MATHEMATICS

CURRICULUM – SYLLUBUS, ACADEMIC STANDARDS

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Foreword

Change is inevitable. We are living in such a dynamic society which is undergoing rapid changes day by day. Needs of society are also changing Children & Society Education system is not exceptional as the changes are concerned. The needs of children are also changing as per the changes brought out in the society. Therefore, the need for the change in curriculum to address contextually needy.

The main goals of mathematics education in schools is the mathematisation of the child's thinking. Clarity of thought and pursuing assumption to logical conclusions is central to the mathematical Education. There are many ways of thinking and the kind of thinking one learns mathematics is an ability to handle abstractions and an approach to problem solving.

According RTE-2009 act, universalisation of elementary education, in all aspects is focused for 6-14 years age of children. The universalisation of education it has an important implications for mathematics curriculum. Mathematics being a compulsory subject of study at elementary level access to quality mathematics education is necessary for every child.

Mathematics education should be given to every child in an interesting and joyful way. As per as the changes are concerned. The maths curriculum is to be implemented where

1) Children learn to enjoy mathematics.
2) Children learn important mathematics concepts.
3) Mathematics is a part of children's life experience which they talk about.
4) Children pose and solve meaningful problems.
5) Children use abstractions, perceive relationships and structures.
6) Children understand basic structure of mathematics.
7) Teacher engage every child in class, and make them participate in maths tasks.
8) Children integrate mathematics with other subjects and daily life technology.

As per as the above aspects, it is necessary for change in curriculum, syllabus, textbook, teaching learning process and evaluation process in Maths Education. Therefore this hand book discuss not only the nature mathematics but also explains the major changes and reforms in the curriculum.

Analysis of above requirements lead us to recommend

a) Shifting the focuss of mathematics education from achieving "narrow" goals to "higher" goals.
b) Engaging every student with a sense of success while at the same time offering conceptual challenges to the emerging mathematicians.
c) Changing modes of assessment to examine students mathematisation abilities rather than procedural knowledge.
d) Enriching teachers with variety of mathematical resources

Our vision of excellent mathematical education is based on twin premises that "all students can learn mathematics" and "that all students need to learn mathematics". It is therefore imperative that we offer mathematics education of the every highest quality to all children.

In the preparation of this document the guidance of national experts has been taken. We are grateful to them. Many state level mathematical education experts, NGO's, classroom teachers have also participated in the preparations of this document. We are thankful to them. Suggestions are invited from interested teachers and other professionals across the state to improve this edition further.

MATHEMATICS
CURRICULUM, SYLLABUS AND ACADEMIC STANDARDS
Contents

1. MATHEMATICS – SYLLABUS – NEED FOR CHANGES

Introduction

What mathematics? Why it should be learned and when?

Is it only learning of numbers and operations? or any thing else?

Is it learning of tricks to solve problems? or any broader goals & expectations?

Answer for these questions may not be simply! To answer these questions, let's observe Teaching Learning Process in a general classroom. Traditionally learning of mathematics in classroom was mainly focused on memory. Memorising some formulas and some tricks to solve the problems were the key aims in the mathematics classroom. Moreover, teaching of mathematics was focused on Examinations. The students who achieved more marks in exams and who achieved ranks are considered as bright students. Thus teachers and students focus more upon marks rather than developing mathematical skills. This practice has lead to classifying the student such as bright and dull. Here, surprising fact is that most of the students are feeling mathematics as a difficult subject, and they developed misconception that mathematics can be learnt by only some students. Thus, a phobia on mathematics is developed among larger section of student & society. Moreover, we can't say that a student achieved more marks in exams has achieved all the competencies in those concepts.

If we observe the teaching learning process of mathematics in a classroom, it has confined to mere transfer of knowledge (i.e. information). A teacher stands at backboard and speaks to transfer information of the content provided in the textbooks. While teaching of "solving a problem", he teaches the steps to solve problems and the students has to listen it silently. After this, the students have to copy whatever explained on the blackboard.

Can this really develop "problem solving" skill among students?

Children have natural abilities like observation, questioning, exploration, enthusiasm to learn new things, analytic and synthetic etc. There is no place for these skills and utilize in teaching learning process in the classroom

When a person is not allowed to utilize his natural abilities, the abilities will not be nurtured and children do not conceptualise the marks concepts. Thus, the student has to depend upon teacher or elders to learn or solve any problem in mathematics in a rote methods.

Therefore, where does the problem lie?, Where to address & How?

Is there any problem with textbooks?

Is it the attitude of mathematics teachers?

Is there any problem with teaching learning process of mathematics?

Is there any problem with the curriculum?

Let us Examine.

what is curriculum?

According to Kothari commission "Totality of learning experiences provided to students so that they can attain general skills, abilities and knowledge at a variety of learning sites".

And according to Smith. Stanley & shores "Curriculum is a cultural reproduction in a structural way. It is more. It should also value independent thinking in the context of the widest sense of social responsibility.

Hence, when we try to understand the meaning of curriculum,

- It is the sum of total experiences provided by the school in and out side the school.
- It should direct the child to achieve standards.
- It should be useful in his daily life and in development of the society.
- It should realize the child about social responsibility.

Mathematics learning does not mean learning of definitions, formulae and solving problems given in the textbook. Mathematics has its own nature. Mathematics has logic and everything in it is developed based on reasoning. Mathematics classroom should encourage the child to understand and develop logic. A mathematics classroom should provide variety of experiences to the child to think and allow the child to think on the basis of his daily life experiences. It should give space to the child to express his logic and opinions. Finally, the child should understand the concept on his own based on his experiences and observations but he should not be taught everything by the teacher.

Mathematics is queen of all sciences. Mathematics concepts are related to the concepts of other subjects. But, mathematics has been treated as single subject in its own. Though, some people have been treating that it is related to physics. But, mathematics textbooks should encourage the child to relate the mathematical concepts with the concepts in other subjects.

Mathematics textbooks were filled up with abundant information. Most part of the textbook has not allowed the child to think and understand the concepts on his own. The teachers and student are confined to learn whatever the information provided in the textbook. The learning of mathematics has become mere completing syllabus and preparing for exams. Though the child achieves good marks in the exams, it may not guaranty that it is useful to him in his daily life or development of maths abilities. Most of the times we have seen that the children who performed well in exams have not performed well in his daily life in higher level Mahts.

Children are verymuch interested in playing games. They are always ready to accept challenges. Children are always interested to know about new things by direct experiences with objects. These things should become strategies in achieving mathematical aims. Mathematical abilities should be developed in child so as to solve problems of his daily life. A mathematics classroom should develop abilities in the child which are useful in the development of society & further learning maths.

Based upon above discussion, the need has been arised to bring out changes in mathematics curriculum i.e. the textbooks, the teaching learning process and evaluation. Hence, a vision document NCF – 2005 has been prepared by NCERT on the basis of vast research and discussions by expert group by NCERT. It also suggested SCERT's of respective states to prepare their own curriculum frame work documents. Thus APSCERT has prepared APSCF-2011, in which it discussed nature of mathematics, types of mathematics reasonings, academic standards, learning strategies evaluation reforms etc.

In forthcoming chapters, we discuss elaborately about major changes which were brought out in Maths curriculum. Objectives of mathematics, class wise academic standards and syllabus, learning strategies, resources and learning materials in mathematics, innovative strategies for teaching of maths etc and discussed.

2. APSCF – 2011 - EXECUTIVE SUMMERY

Schooling in this country was once a privilege of the few but today it is a fundamental right through Right to Free and Compulsory Education Act, 2009. Providing education to all is an important goal upheld by the Indian Constitution. India took up concerted efforts to establish a system of mass education more than 60 years ago. Constitution of India made it obligatory for the state to provide basic education to all in the age group of 6 to 14. This was a huge task. India is a nation of diversities with varied cultural and linguistic pluralities. It is also a nation committed to democratic values and social justice. Andhra Pradesh is no exception to this. The AP SCF is in agreement with the principle stated in NCF 2005 that the child should be at the basis of the education system. However, in addition, AP SCF takes the stand that learning should be interaction based, and that interactions between different components of the system should be given equal importance. This principle will guide our perspectives and actions in the realm of education.

As NCF 2005 states, what we include in 'knowledge' reflects our opinion of what we think the aims of education are. Since our aims of education are to enable the child to think critically, to observe, to analyze, rationalize and draw patterns, and to be sensitive to the diversity or human life, 'knowledge' should not only include pieces of information, but ways of thinking and feeling. We often think of knowledge as information that a child should simply acquire by repeatedly reciting or memorizing, but knowledge includes many more things other than knowing trivia about the world. The aim of education, therefore, is not to feed the child with pieces of information, but to hone her thinking skills. In order to achieve this, children will need sensitive scaffolding at appropriate moments and will have to play an active part in creating that knowledge for themselves and to analyse it.

In ensuring that this happens, the role of the teacher is very important. In spite of all the technological breakthroughs and the arrival of the virtual classroom, the importance of the classroom teacher has not reduced. In the Indian culture, the teacher has been given great position and respect. Although times have changed since, most Indians still look at the teachers with the same respect and awe. However, a shift in the traditional role of the teacher is recommended here. Teacher should act as facilitators of knowledge rather than givers of knowledge. They should regard their students as constructors of knowledge rather than mere recipients thereof; and should have positive attitudes towards the learner as well as the learning process.

Along with the interactions between the child, teachers, parents and the community, interactions must also take place between the child and the learning resources. AP SCF believes that there is a set of underlying cognitive abilities such as analytical skills, logical reasoning and inference building which in different forms underlie all system of knowledge. After completion of school education children should develop a scientific temper, specific attitudes, physical skills, language abilities and abstract thinking. In addition to that children should be able to appreciate diversities in the society with a humanitarian perspective, and to think critically and creatively. They should become responsible citizens and rational human beings. The knowledge that is generated from the school must be based on child background and their experiences. Crafts, arts, play, work, health are also key areas in school curriculum along with subject specific areas. Assessment is also an integral part of the learning process and of ensuring quality education.

Quality education also implies providing children with an environment conducive to learning in schools. This goes beyond the teaching process and the curriculum to the physical environment and resources available to the child. Unfortunately, even today, in many cases remote isolated habitations, girls, minorities and children with disability do not have access to school. Even those who do have access, the quality of educational environment is often poor. This often acts as a deterrent to the educational process. This environment needs to be examined and reformed wherever required.

Along with the learning environment, there is also a need for systemic reforms. There are several problems in making the different components involved in the education of children work together in harmony but unless that happens, the project of quality education for all may remain a dream. We need to make changes towards a resolution of various problems which characterise the system. These need to be related to changes in the curriculum, in attitudes and preparation of teachers, in the support structures for teachers and others engaged in educational efforts, as well as in the nature of relationship between the community and the educational institutions. It is only through enhancing the quality of interactions between various agencies and stakeholders that education can be made interactive and meaningful.

Guiding Principles

What should be the guiding principles for the AP State Curriculum Framework? We may first turn to NCF 2005, RTE 2009 and the recommendations of the National Knowledge Commission.

National Curriculum Framework 2005

The National Curriculum Framework - 2005 is a means of modernizing the system of education. Following are the guiding principles as proposed by NCF-2005.

- connecting knowledge to life outside the school,

- ensuring that learning is shifted away from rote methods,
- enriching the curriculum to provide for overall development of children rather than remain textbook centric,
- making examinations more flexible and integrated into classroom life and,
- nurturing an over-riding identity informed by caring concerns within the democratic polity of the country.

Right to Free and Compulsory Education Act 2009

The RTE Act provides free and compulsory education as a right to every child in the age group of 6 to 14 years in a neighborhood school till completion of elementary education. No child can be refused admission on any grounds and will be admitted to her age appropriate class and make suitable arrangements so that the child can cope with the current curriculum. Education will be inclusive and the schools are supposed to make appropriate arrangements for children with disabilities and with special needs. The child is not liable to pay any kind of fee or charges and expenses which may prevent him or her from pursuing and completing the elementary education. A key feature of RTE is that it focuses on quality as an integral aspect of the child's right to be educated. Part Vth of the RTE act lays down fairly specific terms under which the quality of elementary education is ensured. The RTE emphasizes that the curriculum and the evaluation procedure for elementary education shall be laid down by an academic authority to be specified by the appropriate Government, by notification. The academic authority, while laying down the curriculum and the evaluation procedure shall take into consideration the following:

(a) conformity with the values enshrined in the constitution;

(b) all round development of the child;

(c) building up child's knowledge, potentiality and talent;

(d) development of physical and mental abilities to the fullest extent;

(e) learning through activities, discovery and exploration in a child friendly and child centered manner;

(f) medium of instructions shall, as far as practicable, be in child's mother tongue;

(g) making the child free of fear, trauma and anxiety and helping the child to express views freely;

(h) comprehensive and continuous evaluation of child's understanding of knowledge and his or her ability to apply the same;

(i) No child shall be required to pass any Board examination till completion of elementary education.

(j) Every child completing his elementary education shall be awarded a certificate in such form and in such manner, as may be prescribed.

Therefore it is mandatory on the part of the state government to take up curricular and evaluation reforms reflected in the above suggestions. The SCERT as academic authority must take up academic reforms as part of implementation of RTE. The proposed curriculum and textbook shall address pedagogic concern articulated in the RTE act.

Constitutional Provisions

- The 73rd Constitutional Amendment created the systems of *gram, taluk and zila* levels to ensure greater participation in the *panchayati raj* in an attempt to give more powers to the local bodies.
- The 74th Constitutional Amendment ensured greater representation of SCs, STs and women on the local bodies.

National Knowledge Commission Recommendations

- Flexibility and autonomy of local level management – the village panchayats must be given the power and autonomy to manage the elementary education as the spirit of 73 & 74 Constitutional Amendments.
- Management of Private Schools
- Focus on Early Childhood Education in view of 0 – 5 years are crucial for learning.
- Administration of School Education Departments and accountability.
- Effective mechanism on monitoring the quality of schools and making schools accountable primarily to the community.
- Social Audit of School Performance.
- Strong Mechanisms and programmes for professional development of teachers and on job support.
- Improved School leadership for managing schools
- Professional sharing and exchange between the schools
- Building of Strong Curriculum Groups and Textbook writers and promote curriculum action research.
- Education of marginalized groups, respect of diversity and equity.

These guidelines rightly lay great emphasis on teacher education, professional training, community participation, teaching-learning process and assessment procedures. We may discuss each one of these briefly.

The National Curriculum Framework for Teacher Education 2010

The NCFTE -2010 focuses on:

- Effective pre-service teacher education
- Strategies for the professional development of in-service teachers.
- Focus on research on curriculum implementation and studies on programme evaluation.
- Professional ethics and teacher preparation.

State Curriculum Framework

Perspective

1. The aims of education should never be lost sight of. The primary purpose of education is to produce rational and responsible citizens who can appreciate their heritage and also become agents of social change.
2. The needs and aspirations of the learner are central to the process of curriculum formation.
3. There is a certain cognitive sequence in learning. The curriculum should be in consonance with the cognitive levels of children. The curriculum should focus more on the process rather than the product. This will help the child to develop understanding rather than just accumulate information. It is also likely to equip the child with analytical skills.
4. Knowledge in terms of basic cognitive abilities needed is in a sense unified. Its division into different 'subjects' is in some sense artificial. The same text can often be used for many purposes such as developing language skills, mathematical abilities or social awareness; it can also be used for logical thinking, analytical skills and inferencing.
5. The curriculum should be dynamic. It should not be confined to the prescribed textbooks only. It must embrace the world outside the school as well as the creativity of the child and the teacher.
6. Decentralisation of all aspects including academic work and administration should be at heart of all educational activities in the State.

What is Curriculum framework?

It is a plan that interprets educational aims vis-a-vis both individual and society to arrive at an understanding of the kinds of learning experiences schools must provide to children. The curriculum framework document provides direction to take up various educational activities, development of syllabus and textbooks etc.

Curriculum is a set of planned activities which are designed to implement particular educational aim – set of such aims in terms of the content of what is to be taught and the knowledge, skills and attitudes which are to be deliberately fostered, together with statement of criteria for selection of content, and choices in methods, materials and evaluation.

The curriculum is a source of everything that is done in classrooms and schools towards children's education. It tells us what is worth teaching, how much should be taught and in what sequence, with what methods and materials, how learning should be assessed, teachers prepared and schools monitored. Curriculum is the source of all works related to education.

What is Syllabus?

Syllabus refers to the content of what is to be taught and the knowledge, skills and attitudes which are to be deliberately fostered with state specific objectives.

Process of Developing SCF -2011

SCERT is expected to review school curriculum as a regular activity ensuring the highest standards of rigour. National Policy of Education 1986, National Curriculum Framework 2005 and Right to Free and Compulsory Education Act 2009 assigns a special academic role to SCERT in preparing and promoting State Curriculum Framework. As part of development of State Curriculum Framework, the curriculum committee examined the major challenges and concerns being faced by the school education system in the state. A high-powered Advisory Committee was constituted. It was decided to develop a Curriculum Framework document along with 20 Position Papers in different domains of knowledge. National and State level experts from different universities and institutions and teachers, teacher educators and NGOs were involved in the process. Huge curriculum load in terms of information loaded textbooks, ineffective methods of teaching learning processes, memory based examinations etc. warranted for improving the existing situation by way of undertaking curricular and examination reforms. This document lays the foundations of a completely fresh perspective on the education of children keeping their potential to learn at the heart of curriculum planning.

SCF 2011: Key Principles

It is required to focus on systemic factors that will address major assumptions, beliefs and attitudes in the system and improve the educational practice with appropriate transformation. The committee formulated the key principles for state curriculum framework. In this regard the guiding principles formulated under NCF-2005 were considered in addition to certain other principles to address the existing challenges. Following are the key principles of the A.P. state curriculum framework.

- keeping the potential of the child to learn always in focus,
- respecting the systems of knowledge such as languages children bring to school,
- connecting knowledge to life outside the school; children should not feel that
- ensuring that learning is shifted away from rote methods and the focus should be on interactions, project work, analysis etc. what they are learning at school has no relevance to their lives,
- enriching the curriculum to provide for overall development of children rather than remain textbook centric,
- making examinations more flexible and integrated into classroom life; more focus on assessment for learning than assessment of learning,
- promoting social constructivism, issue-based curriculum and critical pedagogy across curricular areas,
- nurturing towards flora and fauna and respect for bio-diversity and social diversity, respect to the work shall be promoted as a part of school curriculum,
- locating classroom practices in the languages and cultures of children.

3. MATHEMATICS CURRICULUM

NATURE OF MATHEMATICS – CHILD LEARNING

Introduction

Mathematics is the result of the human mind's ability to abstract from life. It consists of ideas and concepts like numbers which though created for counting have no relationship to the individual characteristics of the objects being counted. Mathematics uses both deductive and inductive reasoning to build itself. What are these? When statements or propositions are based on a set of observations and experiences, drawing on patterns observed and generalizing from them, it is called inductive reasoning. On the other hand when truth is established through the process of deductive reasoning, it is based on a previously established statement and logic.

Why Maths?

For some the value of Mathematics lies in its practical utility i.e. in its application in day-to-day life and work. Others appreciate it as a tool for improving thinking and for some others it is simply pursued because they enjoy it. All three of them are valid reasons for engaging with the discipline and have implications for teaching mathematics in school. By including mathematics in the school subjects we want children to develop skill and understanding in the various curricular areas related to number and space and logical thinking. They should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations.

Maths Vs. Nature of the Child

Children have many innate abilities for Mathematics such as those of classification, matching, estimation, analysis, mapping and generalization. The conceptions of number and space- more-less, far-near, big-small, in-out, tall-short, heavy-light etc. should be utilized in introducing them to formal Mathematics. Children seek to make sense of school Mathematics in their lives. Thus, any attempt to develop teaching learning material for children or engage with them in the classroom must respect their everyday life experiences. In many poor urban households, children participate in economic activities. Almost no school curriculum gives any place to such everyday 'street Mathematics'. Beginning from these concrete experiences children can be helped to move to more abstract mathematics. Mathematics suffers from the syndrome of one correct answer. Children need to be encouraged to come up with and use more than one way/method of doing things and solving problems.

Ability of the child to do Maths

- All children are capable of learning Mathematics.
- All children are curious by nature and interested in learning. They ask about how things happen and why they happen.
- Children learn in a variety ways- through experience, making and doing things, experimentation, reading, discussion, asking questions, observation, learning, playing with puzzles, thinking and reflecting, expressing oneself in speech, movement and writing.
- Children do not only learn individually but also in their interactions with other children.
- Children do not come to school as blank slates. Children have many innate abilities for Mathematics such as those of classification, matching, estimation, analysis, mapping and generalization. The conceptions of number and space- more-less, far-near, big-small, in-out, tall-short, heavy-light etc. should be utilized in introducing them to formal Mathematics.
- Children seek to make sense of school Mathematics in their lives. Thus, any attempt to develop teaching learning material for children or engage with them in the classroom must respect their everyday life experiences.

Objectives of Maths Teaching

For some the value of Mathematics lies its practical utility i.e. in its application in day-to-day life and work. Others appreciate it as a tool for improving thinking and for some others it is simply pursued because they enjoy it. Keeping all this in sight, Mathematics teaching has the following objectives -

- Children should be able to develop skill and understanding in the various curricular areas related to number and space.
- Children should be able to reason mathematically.
- Children should be able to pursue assumptions to their logical conclusion
- Children should be able to handle abstraction.
- Children should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations)
- Children should develop confidence in using Mathematics meaningfully.
- Children should develop the following problem solving capacities.
    - Understand problem / arrange information about the problem.
    - Formulate different tactics for the given problem.
    - Analyse and explain results.
    - Apply those results for new situations or problems or able to generalize.
- Children should have confidence on applying mathematics in a meaningful ways.

Vision for School Maths

- Children learn to enjoy Mathematics rather than fear it.
- Children learn that Mathematics is much more than formulae and mechanical procedures.
- Children see Mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements.
- Children understand the basic structure of Mathematics: arithmetic, algebra, geometry and trigonometry. The basic content areas of school Mathematics, all offer a methodology for abstraction, structruration and generalization.
- Teachers engage every child in class with the conviction that everyone can learn Mathematics.

Engaging Children to enhance Learning

Mathematics is a part of the daily life experiences for all of us. In playing cricket or any other sports children follow many rules and conditions when different situations arise and use them as a whole. They prove, disprove and argue giving reasons with established rules. In Andhra Pradesh girls create different patterns in front of their houses using lines, triangles and closed curves with different symmetries. Children use estimation, problem solving, approximation, reasoning skills in shopping. Making these experiences a part of classroom teaching lies at the core of 'mathematisation of the child's thinking'.

Also, it is often the case that people who are not formally educated or never enrolled in schools use many modes of mental Mathematics. For e.g. children of parents who work as masons, plumbers, cobblers, tailors and artisans have a mathematical learning environment in their houses. A mason digs a well without knowing formal Mathematics. What may be called 'folk algorithms' exist for not only mentally performing number operations but also for measurements, estimation and understanding of shapes and aesthetics? If children are motivated to explore linkages between school and life situations, a natural mathematization process can take root in them.

The learning environment in the school must also give space to the whole range of mathematical processes- problem solving, use of heuristics, estimation and approximation, use of patterns, visualization, representation, reasoning and proof, making connections and mathematical communication.

- Problem posing and solving. Real life problems can also be analyzed in a group and more ideas incorporated as a result of group discussion.
- Finding algorithms in Mathematics. Testing algorithms. Finding more than one way/method of doing things.
- Making conjectures, building arguments, testing them, generalize them and verifying results.
- Mathematical quizzes are a good medium for sparking interest in problem solving. These could be within a school or across schools in the form of a competition. The questions asked should be based upon the syllabus.
- In the classroom students should be encouraged to present ideas, prepare talks and deliver them in front of other students and teachers. The topics may be from the regular Mathematics syllabus and also from the world of science and applications.
- The formation of a Mathematics Club in a school can help create a stimulating mathematical environment in the school.
- A Problem Corner can be started by the Mathematics Club, with suitable puzzles and mathematical problems at all levels.
- Mathematical laboratories can have models of different kinds including geometrical shapes and solid objects like spheres, cubes, etc; charts of interesting curves; biographies of mathematicians; computers with Mathematics software; etc. Posters, charts, equipment for explaining theorems or making measurements can be kept in this laboratory. These have to be supplementary and used as temporary scaffolds to help build abstract concepts.
- Projects involving exploration. For e.g. collecting figures, photographs and models which have mathematical significance, from temples, mosques, churches, wall decorations etc, especially during festivals. Looking for patterns in nature.
- Mathematics teaching can be made more interesting by telling students about the lives and works of some Mathematicians and relating the evolution of Mathematics to historical events. The story of development of various mathematical ideas and concepts can be very interesting and inspiring for children. Audio-visual displays can be prepared which will bring out the latent creativity of children, and the children can also do skits connected with historical themes.

Teaching Learning Resources and Materials

Many resources and material are useful in teaching learning process such as textbooks, teaching learning material, math kit, information and communication technology (ICT), audio-visual aids and workbooks.

- Textbook is the primary resource to the teacher and as well as the child. Teachers use it as a resource to plan lessons, understand concepts and evaluation procedures. Children use the textbook to engage with both concepts and processes.
- Workbooks for students and handbooks for teachers can be given for understanding and applying knowledge in many more contexts and various forms.
- Audio Visual aids may be used by the teachers in transformation of knowledge in a systematic presentation; for example Bulletin board, GeoBoard, Mathematics kit etc.

Textbook Development - Principles

- Textbooks should reflect the experiences of children. Situations that children encounter in real life should be used as far as possible for introducing concepts.
- Textbooks should be able to establish continuity with what children have previously learnt in the topic.
- Textbooks should use simple and unambiguous language. As far as possible they should act as self learning material for the student.
- Textbooks must be inviting in appearance and well structured.
- Each concept and process should be given with examples and exercises. Children should be given practice (in both concepts and process) in various contexts.
- Wherever possible solve problems using more than one method. Encourage children to do the same and also come up with their own ways of solving problems.
- All proofs need to be given in a non-didactic manner, allowing the student to see the flow of reason.

Role of the Teacher

Teacher is an important person in whole education system. Particularly in Mathematics he / she should trust the children abilities and playing vital role in creating attractive classrooms for Maths learning. Teacher may not teach as traditionally what is being done at presently. He / she should be a collaborative learner and make the children to think, understand the concepts and formulate his own rules. It is very important that utilization of available resources particularly locally available materials like pebbles, leaves, stones, puzzles and other materials in teaching learning processes in interactive way.

4. OBJECTIVES OF TEACHING MATHEMATICS

Mathematics is all around us and we use Mathematics everyday but are seldom conscious of this. The patterns in flowers and leaves, the path taken by a buzzing fly, the shape of a matchbox or a building, cooking in the kitchen, playing in the field etc all require mathematical ideas and thinking. In many poor urban households, children participate in economic activities. In a many social and geographical contexts, one finds children engaging with Mathematics outside school. Almost no school curriculum gives any place to such everyday ‘street Mathematics’. At best there may be an attempt to add some contextual details to enhance the 'interest' of children. Thus the Mathematics that child learns to do inside and outside school remaining separate and disconnected. Of course, the larger issue here is of the relation between the school curriculum, and life outside school. Since Mathematics is an abstract branch of knowledge, one may think that there is little to be said about its connecting with culture and everyday life.

Maths at Elementary Level

- Any curriculum for elementary Mathematics must incorporate the progression from the concrete to the abstract. Starting with concrete experiences helps the child understand the connections between the logical functioning of their everyday lives to that of mathematical thinking. At the same time, there is a need to help children handle abstraction.
- Children need to be given space for problem solving, especially problems which present real life situations. They need to be encouraged to come up with and use more than one way/method of doing things.
- Mathematics games, puzzles and stories involving numbers are useful to enable children to make these connections and build upon their everyday understanding. Games – not be confused with open ended play – provide non-didactic feedback to the child, with a minimum amount of teacher intervention.
- The Mathematics syllabus for the elementary classes has to revolve around understanding and using numbers and the system of numbers, understanding shapes and spatial relations, measurement, handling data etc. In this the identification of patterns is central as it helps children make the transition from arithmetic to algebra.

Mathematics at High School Level

- "Skills are taught, concepts are caught." (This has been said by many Mathematics educators, in particular by P K Srinivasan.)
- At this stage Mathematics comes to the student as an academic discipline. At the elementary stage, Mathematics education must be guided more by the logic of child psychology than by the logic of Mathematics. But by the secondary stage, the student begins to perceive the logical structure of Mathematics. The notions of argumentation and proof become central.
- The student needs to integrate the many techniques of Mathematics into a problem solving ability. For e.g. this implies a need for posing problems to students which involve more than one content area: algebra and trigonometry, geometry and mensuration and so on. Trigonometry is used to prove results in Euclidean geometry; for example the theorem of Apollonius. (Note however that it is *not* possible to prove Pythagoras's theorem using trigonometry, as this involves circular reasoning. The student needs to be aware of such logical traps, which are quite common).
- A graded exposure to non-routine problems is essential, right from the early years. This exposure must be gradual. It is pointless either to pose problems that are beyond a child's ability, or so simple that they do not in any way challenge the child. Only a teacher who has worked with the children and knows them well would be able to decide the right level of problems to pose.
- An emphasis on experimentation and exploration is worthwhile. Mathematics laboratories are a recent phenomenon which will expand in the future. Activities in practical Mathematics help students greatly in visualization.

Managing a Mathematics classroom also requires acknowledging that various forms of social discrimination also work in the context of Mathematics education. Gendered attitudes consider 'Mathematics unimportant for specific categories of children' or 'girls incapable of learning Mathematics'. Similar beliefs exist for children belonging to certain castes. Both these need to be challenged in the classroom. Children with special needs, especially children with physical and mental challenges have as much right as every other child to learn Mathematics and their needs have to be addressed seriously. Thus, inclusion is a fundamental part of classroom management. Decisions with respect to the teaching learning methods and materials that a teacher utilizes in the classroom are integral in classroom management decisions.

5. Mathematics Syllabus – Academic Standards - Primary

a. Syllabus - Classes I to V

Class I	Class II	Class III	Class IV	Class V
Geometry (10 hrs.) SHAPES & SPATIAL UNDERSTANDING • Develops and uses vocabulary of spatial relationship (Top, Bottom, On, Under, Inside, Outside, Above, Below, Near, Far, Before, After) SOLIDS AROUND US • Collects objects from the surroundings having different sizes and shapes like pebbles, boxes, balls, cones, pipes, etc. • Sorts, Classifies and describes the objects on the basis of faces, edges, shapes, and other observable properties. • Observes and describes the way shapes affect movements like rolling and sliding. • Sorts 2 - D shapes	**Geometry (13 hrs.)** SHAPES & SPATIAL UNDERSTANDING *3-D and 2-D Shapes* • Observes objects in the environment and gets a qualitative feel for their geometrical attributes. • Identifies the basic 3-D shapes such as cuboid, cylinder, cone, sphere by their names. • Traces the 2-D outlines (faces) of 3-D objects. • Observes and identifies these 2-D shapes. • Describes intuitively the properties of these 2-D shapes. • Identifies and makes straight lines by folding, using straight edged objects, stretched strings and draws freehand with a scale. • Draws horizontal, vertical and slant lines (free hand). • Draw straight-line with a ruler. • Distinguishes between straight and curved lines. • Identifies objects by observing their shadows.	**Geometry (16 hrs.)** SHAPES & SPATIAL UNDERSTANDING • Identifies the side view, top view, front view of objects. • Study of the net of a cuboid and its shape. • Tracing circles, rectangles, squares by using different objects. • Making shapes with matchsticks. • Creates shapes/objects through paper folding and paper cutting. • Identifies 2-D shapes (square, rectangle, triangle, circle) without naming. • Tiles a given region using a given shape. • Distinguishes between shapes that tile and shapes that do not tile.	**Geometry (16 hrs.)** SHAPES & SPATIAL UNDERSTANDING • Identifies the side view, top view, front view of simple objects/ planes. • Identifies of nets of cube and cuboid shaped boxes • Identifies cubes from given nets. • Identifies 2-D shapes viz., rectangle, square, triangle and circle by their names. • Making new shapes/objects using known regular shapes. • Making shapes on the geo-board/ dotted board. • Identifying regular 2D & 3D shapes in objects. • Describes the various 2-D & 3D shapes by identifying and counting their edges, corners and faces. • Draws shapes and patterns - free hand and with scale. • Explores perimeter of simple shapes intuitively and can calculate it. • Explores intuitively the reflections through inkblots, paper cutting and paper folding. • Estimation of area.	**Geometry (16 hrs.)** SHAPES & SPATIAL UNDERSTANDING • Draws the side view, top view, front view of simple objects/ plans. • Makes the shapes of cubes, cuboid using nets especially designed for this purpose. • Uses shapes to create different shapes (tangram) and different patterns • Identifies the shadows of the different given objects. • Identifies appropriate nets for cube and cuboid • Explores intuitively line symmetry in familiar 3-D objects expressed as 2 D shapes. • Explores intuitively rotations and reflections of familiar 2-D shapes. • Explores intuitively the perimeter and area of simple shapes. • Estimates area • Gets the feel of an angle through observation and paper folding. • Identifies right angles in the environment. • Identifies right angle and angles more than and less

Class I	Class II	Class III	Class IV	Class V
			• Measuring area using non standard units • Comparison of big and small non standard units.	than right angles. •Draws right angle and angles more than and less than right angles. • Division of complete angles into parts • Drawing shapes on dotted paper • Identifies and reads floor maps, routes/road maps • Draws simple floor maps of familiar locations • Point, line, vertex, ray • Identifies centre and radius of a circle.
Numbers (46 hrs.) **Developing a sense of Numberness, Counting and Operations of Numbers 1 - 9 and zero** • Observes object and makes collections of objects. • Arranges the collection of objects in order by – One to one correspondence – Matching • Introduction of number(1-5) • Counts the number of objects in a collection. • Makes collection of objects corresponding to a specific number. • Introduction of numbers (6-9) • Recognizes and speaks numbers from 1 to 9. • Uses numbers from 1 to 9 in counting and	**Numbers (46 hrs.)** • Reads and writes numerals for numbers up to ninety nine. • Counts and regroups objects into tens and ones. • Expands a number with respect to place values. • Uses the concept of place value in the comparison of numbers. • Counts in various ways: – Starting from any number. – Group counting etc. • Arranges numbers upto hundred in ascending and descending order. • Forms the greatest and the smallest two digit numbers with and without repetition of given digits. • Indicates and identifies the position of an object in a	**Numbers (42 hrs.)** **Number upto 1000** • Reads and writes up to 3-digit numbers. • Understands place value in 3-digit numbers. • Expands a number using place value. • Compares numbers. • Forms numbers using given digits. • Estimates numbers **Addition and Subtraction** • Solves addition and subtraction problems in different situations presented through pictures and stories. • Adds and subtracts numbers by writing them vertically and horizontally in the following two cases:	**Numbers** (40 hrs.) Numbers upto 10000 • Using word problems/ contextual situations, reads, writes and compares 2, 3-digit numbers. • Estimating 2, 3- digit numbers using the number line • Understands place value in 3-digit numbers. • Expands a number using place value. • Forms numbers using given digits. • Skip counting in terms of tens, hundreds and thousands **Addition and Subtraction** • Using word problems/ contextual situations for a additions and subtractions up to 999. (compare-combination and comparison types of word	**Numbers (40 hrs.)** **Numbers upto 1,00,000** • Using word problems/ contextual situations, reads, writes and compares 3,4,5-digit numbers. • Understands place value in numbers up to 99,999 • Expands a number using place value. • Forms numbers using given digits. • Skip counting in terms of hundreds, thousands and ten thousands **Addition and Subtraction** • Using word problems/ contextual situations for a additions and subtractions up to 99999. (compare-combination and comparison types of word problems)

Class I	Class II	Class III	Class IV	Class V
comparison. • Reads and writes numerals from 1 to 9. • Adds and subtracts using real objects and pictures. (Sum not to exceed 9 and difference not to go below 1.) • Adds and subtracts the numbers using symbols '+' and '-'. • Approaches zero through the subtraction pattern. **NUMBERS FROM (10 - 20)** • Introduction of 10 • Forms Number sequence from 10 to 20. • Counts objects using these numbers. • divides objects into a group of 10s and single objects. • Develops the vocabulary of group of 'tens' and 'ones'. • Shows the group of tens and ones by drawing. • Counts the number of tens and ones in a given number. • Writes number names ten to nineteen. • Writes numerals for ten to twenty. • Compares numbers upto 20. **ADDITION AND SUBTRACTION (UPTO 20)** • Adds and subtracts numbers upto 20. **NUMBERS FROM 21 - 99**	line. **ADDITION AND SUBTRACTION** • Adds and subtracts two digit numbers by drawing representations of tens and ones without and with regrouping. • Adds zero to a number and subtracts zero from a number. • Observes the commutative property of addition through patterns. • Solves addition, subtraction problems presented through pictures and verbal description. • Describes orally the situations that correspond to the given addition and subtraction facts. • Estimates the result of addition and subtraction and compares the result with another given number. (Based on place values.) **PREPARATION FOR MULTIPLICATION AND DIVISION** • Discussion of situations involving repeated addition and situations involving equal sharing. • Activities of making equal groups.	– without regrouping. – with regrouping. • Introduction to different strategies of addition and subtraction • Uses the place value in standard algorithm of addition and subtraction. • Frames problems for addition and subtraction facts. • Estimates the sum of, and difference between, two given numbers. **MULTIPLICATION** • Explains the meaning of multiplication (as repeated addition). • Identifies the sign of multiplication. • Constructs the multiplication tables of 2, 3, 4, 5 and 10 • Uses multiplication facts in situations. • Construct tables for 6, 7, 8, 9 • Multiplies two digit numbers by single digit number using standard algorithm and Lattice multiplication algorithm. **DIVISION** • Explains the meaning of division from context of equal grouping and sharing. • Relates division with	problems) • Using word problems/ contextual situations for addition of 3 numbers. • Estimates sums and differences of 2,3 digit numbers through word problems and in sums. • Adds and subtracts 2,3-digit numbers using the empty number line. • Frames word problems. **MULTIPLICATION** • Using word problems/ contextual situations revises multiplication facts up to 10*10.(array - rate product and grouping types of word problems) • Multiply by 10's and 100's • Using word problems/ contextual situations multiplies 2 and 3 digit numbers by single digit and two digit numbers using the standard (column) algorithm as well as the distributive law. .(array product - rate product and grouping types of word problems) • Frames word problems. • Estimates products 2 digit by 1 digit and 2 digit, 3 digit by 1 digit **DIVISION**	• Estimates sums and differences of 3,4 digit numbers through word problems and in sums. • Frames word problems. **MULTIPLICATION** • Multiply by 10's, 100's, 1000's and 10,000s • Using word problems/ contextual situations multiplies 3 digit number by 2 digit numbers using the standard (column) algorithm as well as the distributive law. (array product - rate product and grouping – Cartesian product types of word problems) • Frames word problems. • Estimates products of 3 digit by 1 digit and 3 digit by 2 digit numbers **DIVISION** • Using word problems/ contextual situations dividing 2-digit numbers by two digit numbers and three digit numbers by two digit numbers- with remainder and without remainder (using both equal grouping and sharing) • Understands the pattern which emerges from division by 10 • Uses standard division

Class I	Class II	Class III	Class IV	Class V
• Writes numerals for Twenty-one to Ninety nine.· • Groups objects into tens and ones. • Draws representation for groups of ten and ones. • Groups a number orally into tens and ones.		multiplication. • Completes division facts: (Double digit by single digit) – by repeated subtraction – by grouping – by using multiplication tables.	• Using word problems/ contextual situations dividing 2 and 3 digit numbers by one and 2-digit numbers - with remainder and without remainder (using both equal grouping and sharing) • Frames word problems. • Even and odd numbers • Estimates quotients for 2 and 3 digit numbers divided by single digit numbers. • Explores the relationship between multiplication and division using 2 and 1 digit numbers	algorithms for two-and three digit numbers divided by one and two-digit numbers • Frames word problems. • Even and odd numbers • Tests of divisibility for 2, 5 & 10. • Understanding of the multiples and factors • Estimates quotients • Explores the relationship between multiplication and division using 2 and 3 digit numbers
			FRACTIONAL NUMBERS • Identifies half, one fourth and three - fourths of a whole. • Identifies the symbols, ½ , ¼ , ¾ . • Explains the meaning of ½ , ¼ and ¾ . • Identifies other fractions- 3/2, 5/2, 5/4 • Appreciates equivalence of 2/4 and ½ ; and of 2/2 , 3/3, 4/4 and 1. • Comparison of like fractions • Addition and subtraction of like fractions intuitively	**FRACTIONAL NUMBERS** • Finds the fractional part of a collection/ object • Identifies equivalent fractions 2/4 and ½ ; 2/6 and 1/3, 2/8 and 1/4 • Compares like and unlike fractions(without LCM) • Addition and subtraction of like fractions • Applies simple fractions to measurements.
Day to Day Maths (3 hrs.) (Money, Length, Weight, Capacity) • Identifies common currency notes and coins.	**Day to Day Maths (3 hrs.) (Money, Length, Weight, Capacity)** • Identifies currency - notes and coins.	**Day to Day Maths (5 hrs.) (Money, Length, Weight, Capacity, Space)** • word problems/ contextual situations using more than	**Day to Day Maths (5 hrs.) (Money, Length, Weight, Capacity, Space)** • word problems/ contextual situations using more than	**Day to Day Maths (5 hrs.) (Money, Length, Weight, Capacity, Space)** • word problems/ contextual situations using more than

Class I	Class II	Class III	Class IV	Class V
• Puts together small amounts of money.	• Puts together amounts of money not exceeding Rs 10/- or Rs. 50/-. • Adds and subtracts small amounts of money mentally. • Transacts an amount using 3-4 notes/coins.	one operations and/ or more than one concept and/or multiple stages of solving • estimation in daily life	one operations and/ or more than one concept and/or multiple stages of solving • estimation in daily life	one operations and/ or more than one concept and/or multiple stages of solving • estimation in daily life
Measurement (13 hrs.) **(LENGTH, WEIGHT, CAPACITY)** **LENGTH:** • Distinguishes between near, far, thin, thick, longer/taller, shorter, high, low. • Seriates objects by comparing their length. • Measures short lengths in terms of non-standard units (e.g. hand span etc.) **WEIGHT** • Compares between heavy and light objects.	**Measurement (13 hrs.)** **(LENGTH, WEIGHT, CAPACITY)** **LENGTH:** • Measures lengths & distances along short & long paths using uniform non-standard units (Foot). **WEIGHT** • Compares two or more objects by their weight using non-standard units • Appreciates the need for a simple balance. • Compares weights of given objects using simple balance	**Measurement (21 hrs.)** **(LENGTH, WEIGHT, CAPACITY)** **LENGTH:** • Appreciates the need for a standard unit. • Measures length using appropriate standard units of length by choosing centimeters. • Compares the length of given objects in standard units and verifies by measuring. • Uses a scale for measuring **WEIGHT** • Weighs objects using 1kg. • Estimates the weight of an object and verifies using a balance. **CAPACITY** • Measures and compares the capacity of different containers in terms of nonstandard units. • Appreciates the conservation of capacity. • Solves the problems on 'capacity'(in non standard units)	**Measurement (21 hrs.)** **(LENG,TH WEIGHT, CAPACITY)** **LENGTH:** • Identifies meter and cm lengths • Relates meter with cm. • Converts meter into cm. • Measures length in meters, cm's and inches. • Estimates length of an object and distance between two given locations. • Solves problems involving length and distances in *m* and *cm*. **WEIGHT** • Understands weight in terms of *kg* and *g*, using actual weights and their combinations • Relates Kg with gram • Weighs objects using a balance and standard units. • Appreciates the conservation of weight. • Estimates the weight of an object. Verifies using a balance. • Solves problems involving weights using *kg* and *g*.	**Measurement (26 hrs.)** **(LENGTH, WEIGHT, CAPACITY)** • Relates commonly used larger and smaller units of length, weight and capacity and converts one to the other. • Relates feet to inches. • Relates km to m; liter-ml; kg-gram; quintal-kg • Applies simple fractions to quantities. • Converts fractional larger unit into complete smaller units. • Applies the four operations in solving problems involving length, weight and capacity. • Determines intuitively area and perimeter. • Estimated length, weight, capacity of a solid body: intuitively and also by informal measurement. • Understands the concept of area

Class I	Class II	Class III	Class IV	Class V
			Capacity • Understands capacity in terms of *l* and *ml* • Relates liter with ml. • Measures capacity of given liquid using containers marked with standard units. • Estimates the capacity of a liquid .Verifies by measuring. • Determines sums and differences of capacity. • Solves problems involving capacity in *l* and *ml*	
Time • Distinguishes between events occurring in time using terms -earlier and later. • Gets the qualitative feel of long & short duration, of school days v/s holidays. • Narrates the sequence of events in a day.	**Time** • Gets familiar with the days of the week and months of the year. • Sequences the events occurring over longer periods in terms of dates/days.	**Time** • Reads a calendar to find a particular day and date. • Reads the time correct to the hour. • Sequences the events chronologically. • Compares the duration of two different events. • Identifies the patterns in a calendar.	**Time** • Appreciates the difference in time in terms of minutes, hours, days, weeks and months. • Reads the calendar, identifies and correlates the number of days, weeks, months and years. • Understands a leap year. • Reads clock time to the hours and minutes • Solves problems on ‘time’	**Time** • Appreciates the difference in time in terms of seconds, minutes, hours, days, months and years. • Reading time in hour, minutes and seconds • Converts hours into minutes and minutes into seconds • Expresses time, using the terms, ‘a.m.’ and ‘p.m.’ • Understanding 24 hour clock (Bus and Railway timetable) and conversion between 12 hour and 24 hour clocks • Computes the number of days between two dates. • Uses addition and subtraction in finding time intervals in simple cases.

Class I	Class II	Class III	Class IV	Class V
Data Handling (6 hrs.) • Collects data • Records data in a table • Draws inferences from data.	**Data Handling (6 hrs.)** • Collects data • Records data in a table • Draws inferences from data.	**Data Handling (6 hrs.)** • Collects data • Records data in a table •Reads data given in the form of a pictograph • Records data using tally marks • Draws conclusions from the data.	**Data Handling (6 hrs.)** • Reading data using a pictograph • Reading data using a bar graph • Recording data using tally marks	**Data Handling (6 hrs.)** •Reading data using a pictograph •Understands the importance of an appropriate scale for pictograph •Reading data using bar graphs • Organizing data using tally marks using tally marker. •Making bar graphs based on data(without scale)
Patterns (10 hrs.) • Describes sequences of simple patterns found in shapes in the surroundings and in numbers, e.g. stamping activity using fingers and thumb. • Completes a given sequence of simple patterns found in shapes in the surroundings and in numbers.	**Patterns (10 hrs.)** • Observes and extends patterns in sequence of shapes and numbers. • Searches for patterns in different ways of splitting a number. • Creates block patterns by stamping thumbprints, leaf prints, vegetable prints, etc. • Creates patterns of regular shapes by stamping.	**Patterns (6 hrs.)** • Identifies simple symmetrical shapes and patterns. • Makes patterns and designs from straight lines and other geometrical shapes. • Identifies patterns in his Surroundings • Symmetry through paper folding, paper cuttings • Complete a symmetric picture using mirror.	**Patterns (6 hrs.)** • Identifies, carries forward and makes visual patterns. • Carries forward patterns in numbers based on addition, subtraction, multiplication and division	**Patterns (6 hrs.)** • Identifies patterns in square numbers, triangular numbers. • Identifies patterns in multiplication and division. • Numbers between consecutive square numbers. • Makes border strip and tiling patterns. • Identifies the patterns in the multiples of 9. • Draws symmetric pictures and symmetric axis. • Identifies the block or unit of the pattern.

b. Maths Academic Standards – Learning Indicators – Primary Level

In the process of education the child not only learns in the classroom but also every where even outside the classroom. Every child has natural abilities to learn, but the education sharpens his abilities and converts those abilities into skills. Whatever child experiences even outside the classroom, the classroom should utilize those experiences to enhance them and help the child to even create new learning's. Speaking mathematically, Mathematics learning should develop skills like problem solving, thinking logically, Reasoning, Representing, connecting etc. Therefore, we need some specific statements which guide us to develop those skills in our mathematics classroom. There statements are named as "Academic Standards".

"Academic standards are clear statements about what students must know and be able to do within a stipulated period to perform skills in a particular content or connecting contents.

To understand about how to write academic standards, we have to understand clearly what are academic standards.

- They are clear statements even normal public should understand them.
- They guide as for the teaching learning process about what skill to be performed by the children after learning.
- They guide us for the assessment of child's performance.
- Sometimes they may be defined for more than one content i.e. combination of contents.
- Sometimes they may be defined with connecting of multiple skills.
- Sometimes they may be defined with multi concepts.

Therefore, we can clearly say that academic standards are clear statements with contain single content or multiple contents, single skill or multiple skills and even with single concept or multiple concepts. Teachers should take the responsibility of performance of the child which is defined by the academic standards.

Now, let us book at the skills to be developed in our mathematics classrooms.

Problem solving :

Usually, students solve problems with a formula, substituting values in it and finding solution. Is this really problem solving skill, what we are expecting from the child? Whatever the mathematics concept is understood, it should be applied in various different situations. The student should think and establish relation with his different experience daily life. Then he seeks ways and means of mathematisation using symbols, processes like addition, subtraction, multiplication, division, squaring etc and gets solution in problem solving. Through problem solving, student gets the pleasure of finding solution when she verifies, gives reasons explains processes, concepts through easy communication, links or connects with different concepts. Mathematics learning should not force the child to find the alternative procedures / ways of finding solutions. When a student is habituated all the process of problem solving and be able to connect it with his daily life, she could create many more different problems with different situation and with different types of numbers.

Therefore to understand problem solving, we need to understand the following steps in problem solving.

Steps in problem solving

- Identify what is given?
- Identify what is to be found?
- Understanding what concepts are involved.
- Visualizing whole the above items.
- Get ideas about procedures, formulas for the solution.
- Selection of the best procedure or formula.
- Substitution.
- Manipulation / calculation.
- Arriving solution.
- Verification.
- Conclusion.
- Generalisation.
- Trying out other strategies, formulas, procedure for the solution.
- Finding shortcut.
- Explaining procedures and reasoning.
- Creating similar problems in various situation and with various types of numbers.

Though the term ‘Problem solving’ may look simple and even the process may look simple some times. There will be problems with more complexity. The amplexity of the problems depends upon the following things.

- Making connections as defined in connections section.
- Number of steps.

- Number of operations.
- Context unraveling.
- Nature of procedures.

There are many types of problems like word problems (with single concept or multiple concepts, with single operation or multiple operations) pictorial problems, procedural problems, Mathematical abstract problems with equations or in equations, reading data, tables, graph etc.

Hence finally we can conclude that problem solving skill in mathematics learning should make the student to think logically, give reasons, connect concepts, visulise the things etc : Mathematisation of child's life would emerge the child into a mathematician.

Reasoning – proof

Every student has potential for higher order – thinking. The key is to unlock the world of mathematics through a students natural inclination to strive for purpose and meaning. Reasoning is fundamental character to the knowing and doing of mathematics. Conjecturing and demonstrating logical validity of conjectures are the essence of the creative act of doing mathematics. Mathematics teacher has been felt so as learn everything from the teacher. This opinion is making the students to completely depend upon teachers have been felt so as learn everything from the teacher. This opinion is making the students to completely depend upon teachers and not allowing students to think on their own, generative and conclude. When a student is allowed to think, generative and conclude on his own, his ability of reasoning is developed. Then the student can give reasons mathematically. When a student understands, analysis the context, make intuitions, conjectures and finally generalizes, then we may that he concluded logically. This may be reflected by his justification of the argument or procedures. Reasoning skills allows a student to examine logical arguments. Most of the mathematical statements are the result of inductive and deductive logics.

The student should perform Reasoning – Proof

- Understanding and making mathematical generalizations, intuitions and conjectures.
- Understanding and justifies procedures.
- Examining logical arguments.
- Uses inductive and deductive logic.

Communication

Is mathematics full of numbers or Is it manipulation of numbers? If you want say 3 objects are more than 2 objects, then what would you like to do?

Communication is an essential part of mathematics and mathematics learning. It is a way of sharing, clarifying, reasoning, generating our understanding. Through communication, ideas become objects of reflection, refinement, discussion and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. When students are challenged to think and give reason about mathematical concepts and to communicate the results of their thinking to others orally or in writing.

The communication skill is reflected in :

- Writing and reading mathematical expressions like $3 + 4 = 7$, $3 \times 4 = 12$, $3 < 4$ etc.
- Creating mathematical expressions.
- Explaining mathematical ideas in her own words. Ex : A square is closed figure having four equal sides and all equal angles.
- Explaining mathematical procedures. Eg: Adding two digit numbers involves first adding the digits in units place and then adding the digits at the tens place / keeping in mind carry over.
- Explaining mathematical logic.

As mathematics learning is carried out in mechanical, way, negligence on mathematics communication prevailed in mathematics classrooms. Wherever it is possible, the students should be allowed to speak on mathematical equations an expressions. For eg: If a student has been allowed to think about $x + 2 = 6$, He should visualize it as the sum of x and 2 gives 6. Then it will be easy for him to solve the equation. Therefore, the student needs communication skills to give proper reasonings or proper conclusions or to solve problems in mathematics.

Connections :

As we discussed earlier, mathematics learning needs to develop logic in the child it helps the child to give reasons for him to conceptualise and solve problems wherever they come across. To develop in conceptualization process, the student has to link or connect things in logical manner one by one, finally generalizes and comes to conclusion. Moreover, If we look at problem solving the student decides a strategy to solve the problem after "making connection" in between the given things in the data of the problem. Therefore, in the process of development of logic or problem solving, "making connections" is an important skill in mathematics learning.

In the process of connecting things in mathematics learning, he needs to connect the abstractions in mathematics with the objects or contexts in his daily life or with concepts in other subjects. He even may need to connect the abstractions with the concepts within the mathematics. This performance of the child will help the child to link mathematics with his daily life. Hence mathematisation of child's life is possible as expected by NCF-2005. Making "connections" is specified by the following performance in the children.

- Connecting concepts within a mathematical domain fore relating adding to multiplication, parts of a whole to a ratio, to division. Patterns and symmetry, measurements and space.

- Making connections with daily life.
- Connecting mathematics to different subjects.
- Connecting concepts of different mathematical domains like data – handling and arithmetic or arithmetic and space.
- Connecting concepts to multiple procedures.

Hence, performance of the child in mathematics can be adjudged by how best the logical connections he is making and arriving at conclusions. These connections may be in between concepts of mathematics or the concept and area or the concept and other subjects or the concept and daily life contexts.

Visualization & representation

We conduct many programs or do several works in our daily life. We visualize a plan or imagine a sequence of actions to make the program successful. Mathematics is involved in almost every program we conduct. For example, we deal with costs of objects, measurements to construct a building as perform a marriage. Like this, mathematical visualization skill is a necessary skill in our daily life.

Visualization creates mental images in the mind when there mental images are related or liked with a logic, visualization about a context or situation or procedure is formed. Hence, the child in our classroom needs visulisation skill in the process of conceptualization or problem solving while mathematics learning. While conceptualization, he forms an idea or notion about a concept by visualizing the thing involved in it. Without visualizing, one cannot understand any concept. Not only in the process of conceptualization, But also in the process of solving problem, the child needs visualization skill. In the process of problem solving, a child needs to visualize all possible strategies to solve a problem and select the best way to solve it. This skill helps the child to develop his logic in mathematics.

When we want to convey our visualization, we represent to in the form of a flow chart or table or graph or any other pictorial form. Representation skill is another import skill to perform in mathematics learning.

Therefore, we need following specifications to perform in "Visualization & Representation".

- Interprets and reads data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures etc.
- Making tables, representing number line, pictures etc.

Hence, Visualization & Representation skill provides easyness to convey our perceptions on ideas.

Till now, we have discussed about the skills on the basis of which we have to write academic standards. When we decide and write an academic standard, it guides our teaching learning process to achieve that standard in the child. Moreover, these academic standards ensure the "performance" of the child and they are displayed in the form of performance.

Academic standards are clear statements about what students must know and be able to do. The following are categories on the basis of which we lay down academic standards.

Content	Problem Solving	Reasoning Proof	Communication	Connections	Visualization & Representation
• Content areas of Maths	Using concepts and procedures to solve. ***a. Kinds of problems:*** Problems can take various forms- puzzles, word problems, pictorial problems, procedural problems, reading data, tables, graphs etc. • Reads problems • Identifies all pieces of information/data • Separates relevant pieces of information • Understanding what concept is involved • Selection of procedure • Solving the problem ***b. Complexity:*** The complexity of a problem is dependent on • Making connections(as defined in the connections section) • Number of steps • Number of operations • Context unraveling • Nature of procedures	• Understanding and making mathematical generalizations, intutions and conjectures • Understands and justifies procedures • Examining logical arguments. • Uses inductive and deductive logic	• Writing and reading mathematical expressions like $3+4=7$ $3\times4=12$ ¾ • Creating mathematical expressions • Explaining mathematical ideas in own words like- a square is closed figure having four equal sides and all equal angles • Explaining mathematical procedures like adding two digit numbers involves first adding the digits in the units place and then adding the digits at the tens place/ keeping in mind carry over. • Explaining mathematical logic	• Connecting concepts within a mathematical domain- for example relating adding to multiplication, parts of a whole to a ratio, to division. Patterns and symmetry, measurements and space • Making connections with daily life • Connecting mathematics to different subjects • Connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space • Connecting concepts to multiple procedures	• Interprets and reads data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures • Making tables, number line, pictures etc.,

c) CLASS WISE ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : I MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Geometry shapes and spatial understanding	• Develops and uses vocabulary of special relation • Slip (top) bottom, on, under, inside, outside, below, near, far, before, after	• Sorts 2D objects using character • Stick of the shapes (edges, faces and other observable)	• Pupils can compare 2-D and 3-D shapes (without mathematical terms) and gives reasons • Observe And describes the way shapes affect moments like rolling and sliding.	• Pupils can observe things and speak about them • They can use terms like bottom, on, under, inside, outside, above, below, near, far, before, after etc.	• Pupils connects the concepts of 2-D and 3-D shapes in understanding them.	• Pupils can draw different 2D shapes • Pupils can represent different shapes with different colours in a given picture.
Numbers	• Developing a sense of numbers, counting and operations of numbers 1-9 and o, introduction of Tens from (10-100) • Numbers 21 to 99	• Pupils can count numbers 1-9 and 10-20 by using things in groups. • Pupils can arrange the things in sequences according to their numbers (Ascending and descending order) • Pupil can add and subtracts using real objects and pictures (sum not to exceed 9) and difference not to go below 1.	• Pupils can compare numbers by counting using things and (1-20). • Pupils can identify things of equal numbers • Pupils can give reasons for their conclusion in comparing numbers.	• Pupils can read and write numbers 1-9, 10-20 and 10-100 (in tens), and 21-99 • Pupils can say the first, middle, previous, after numbers w.r.t particular numbers. • Pupils can say ‘0’ before ‘1’ • Pupils can develops the vocabulary ‘ones’ and tens	• Pupils connect the value of one number with that of another number in arranging them in an order.	• Pupils can represents the numbers 1-9, 10-20 and 10-100 in tens and 21-99 by using thing (marbles, pebbles, beeds etc) • Pupils can show group of tens and ones by drawings.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Pupils can count the number of tens and ones in a given number.				
	• Additions and Subtractions (up to 20)	• Pupils can add and subtract numbers upto 20 (vertically & horizontally) • Pupils can subtract numbers up to 20 not exceeding 20 (vertically and horizontally)	• Pupils verify the result of addition and gives reasons. • Pupils verify the result of subtraction and give reasons.	• They can use symbols +, - and write mathematical form for addition and subtraction (vice-versa).	• Pupils connect "number sense" in under standing addition & subtraction. • Pupils apply addition & subtraction in daily life.	• Represents addition & subtraction in the form of beeds and sticks bundles.
DAY – TO-DAY mathematics	• Money (currency notes and coins)	• Identifying currency notes and coins and can understand the value of each coin and note. • Pupils can put together smaller amounts of money	• Pupil can distinguish and compare difference currency notes and coins.	• Pupils can say the value of a note and coin (Notes 5,10,20,50 coins 1,2,5 and 50 paise)	• Pupils connect number sense in understanding currency & its operation.	• Pupils can represent the value of given money with currency notes and coins.
Measurements	• Length, Weight, size	• Pupil can segregates objects according their lengths • Pupils can measure short lengths in terms of non standard units (span, hand)	• Pupils can compare the lengths of objects and verify. • Pupil can compare between heavy and higher objects. • Pupil can compare the capacity of different vessel (more, less)	• Pupil can estimate and say the length, weight given objects. • Pupils use the terms "morning", "afternoon", "evening" to express time of event.	• Pupils apply concepts of length, weight and time in daily life.	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
TIME	• Time	• Pupil can identify different time of complete day (Morning, daytime, evening, night)		• The pupil can say what people can do at these times. • And they can say what they must do at what time		
Data Handling	• Collecting data • Recording Data • Drawing inferences	• Pupil can collect the data from surroundings and draws inferences from the data	• Pupil can analyse the given data	• Explains about the data		• Pupil can record the data in given space
Patterns	• Finding simple patterns in shapes in the surroundings and also in numbers	• Pupils can find the patterns in shapes and in numbers. • Completes a given sequences of a pattern	• Pupil can give reasons for a sequence of a pattern	• Pupil can describes the pattern.		

Developing children's abilities for mathemtisaation is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'use full' capabilities, particularly those relating to numeracy – numbers, number operations, measurements, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things and the ability and the attitude to formulate and solve problems.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : II MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
GEOMETRY (SHAPES & SPATIAL UNDERSTANDING)	• 3-D and 2-D shapes • Observes objects in the environment and identifies the basic 3-D shapes such as cuboid, cylinder, cone, sphere by their names • Traces the faces of 3-D objects, observes the 2-D outlines and identifies these 2-D shapes. • Identifies and makes straight lines by folding and draws with a scale and ruler. • Distinguishes between straight and curved lines.	• Observes the given diagram colours the same shapes with same colour and counts the shapes	• Can identify odd shapes from given shapes • Can identify number of shapes from the given figure • Can identify the sequence of the simple patterns of figures and carry forward	• Can say the objects or things in environment appropriate 2-D / 3-D shapes.	• Can the interlink the concepts of 2-D shapes to the real objects of their surroundings	• Can join the dots to draw different shapes. • Pupil can represent daily life objects which geometrical shapes.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
NUMBERS	• Understanding numbers upto 999 – can write expansion form, ascending – descending order, can say place of face value. • Adding & subtracting numbers using the carry over borrowing up to 100 • Understanding the multiplication is nothing but repeated addition and make multiplication tab les (1-9) by using repeated addition product of multiplying two digit number by a single digit number • Identifying the symbol of division ( ). Distributing number of things	• Can say the place value & face value of the given 3-digit no's • Can read and write 3-digit numbers • Can write the expended & short forms of three digit No.s • Can write ascending and descending order to the gives 3 digit no.s • Can add the 2-digit numbers carry forward by using expansion and short form. • Can subtract using regrouping borrow of two digit no's • Can make the multiplication on table (1-9) by counting the objects in equal no.of objects in each group or repeated addition.	• Can write the 3-digit numbers by using given no's • Can complete the number series and says the reason • Can identify the sum of one pair is different with the sum of other pairs. • Can estimates the results of addition & subtraction of the 2-digit number and verify it, finding errors and rectify it.	• Can write the 3-digit numbers in words and symbols vice-versa • Can compare the two 3 digits numbers using putting <, >, = symbols. • Can write the multiplication & division form by using symbols to given verbal problems. • Can express the process steps of doing addition, subtraction and also preparing the multiplication tables.	• Can write the repeated addition in to multiplication s and repeated subtraction into division form by using symbols.	• Can observe currency notes coins and write the appropriate numbers • Can represent the given 3-digit number by using 100, 10 currency notes and coins. • Can show the multiplication, Division, addition and subtraction on number line.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can distribute the number of objects in equal				
DAY TO DAY MATHS	• Identifies currency notes and coins. • Puts together amounts of money not exceeding Rs : 50. • Adds and subtracts small amounts of money mentally and transacts an amount using 3 or 4 notes / coins.	• Identifies the currency notes and coins. • Can write the value of the given currency and coins together.	• Can give the change to the value in different ways.		• Can give the value of the coins / currency to the price of the item. • Can connect the number sense in saying the value of currency.	
MEASUREMENT (Length, weight, capacity)	• Measures the lengths & distances using non-standard units.. • Compares two or more objects by using non-standard units. • Compares weights of given objects using simple balance.	• Can measure the lengths of garland, table width, length of the room by using non-standard units like cubit, finger widths, feet etc…	• Can estimate the length by using a stick. • Can estimates the and compares the given two objects.			
TIME	• Gets familier with the days of the week and months of the year.	• Can say the number of days in a week and number of month in a year.		• Can say the activities do in the morning, afternoon, evening	• Can say the activities which takes more / less time in their daily life.	• Can identifies and say the months and weeks by looking the calendar /

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Sequences the events occurring over longer periods in terms of date / days.					chart.
DATA HANDLING	• Collects and records the data and draws inferences from data.	• Pupil can collect the data from surroundings and draw inferences.	• Pupil can analyse the data.	• Can express the recorded information in tables by observation.		• Can record the given data in the table.
PATTERNS	• Observes and extends patterns of shapes and numbers. • Create block patterns by stamping thumbprints, leaf print, vegetable print and regular shapes.		• Can say the relation in between sequence of the given patterns of numbers & shapes. • Can extend the series by reasoning			

Many general tactics of problem solving can be taught progressively during the different stages of school : abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess – and – verify exercises, are useful in many problem solving contexts. Moreover, when children learn a variety it approaches (overtime), their tool kit becomes richer, and they also learn which approach is the best. Children also need exposure, to the use of heuristics or rules of __________, rather than only believing the mathematics is an 'exact science' the estimation of quantities and approximating solutions is also essential skill. When a framer estimates the yield of a particular crop, he uses considerable skills in estimation, approximation and optimization – School mathematics can play a significant role in developing such useful skills.

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ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : III MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Numbers Operations	• Numbers • Additions • Subtractions • Multiplications • Division	• Counts from any number by using groups as 100s, 10s, ones • Can complete given sequences of numbers up to 999 • Able to find the sum of two numbers by joining, combining by grouping, regrouping up to 999. • Can solve the problems of the additions, subtraction of the numbers horizontally, vertically up to 3 digit numbers in different situations (i.e addition – “joining” and “combining” activities) (i.e. is subtraction – activities of “eliminating”, “remaining” (partition) “reduction”, “comparison” and counter addition)	• Estimates the number of objects in a group up to 50. • Compares the numbers upto 999 based on place value • Can form the greatest and smallest two digit and three digit numbers with, and without repetition a given digits. • Determines the reasonableness of calculated answers in addition, subtraction • Creates patterns using numbers involving addition and subtractions up to 50 • Identifies errors in solving addition, subtraction and multiplication • Round the numbers upto the nearest 10s and 100s	• Able to read and write 3 digit numbers (Numbers to words and words to numbers vice versa) • Comparing any 3 digit numbers using symbols (<, >, =) • Pupils can create new problems on their own (addition & subtraction)	• Applies addition subtraction simple multiplications division of 1 digit daily life situations • Uses three digit numbers in daily life (school strength, purchasing articles, pay of workers etc)	• Represents the numbers up to 999 as numbers using cubical blocks.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can multiply two digit b\numbers with one digit number. • Solves the problems on division (division is single digit, without remainder) • Can write the given numbers in ascending, descending orders. • Can expand the numbers according to place values and also write short form at a expanded number.				
Geometry	• Shapes and spatial under standing	• Sorts object using characteristics of shapes • Identify the object by observing different view • Identify basic 2D shapes like square, rectangle, triangle and circle • Distinguishes between the shapes that tile and do not tile • Identifies the 3D objects when we trace, its shapes would be circle, square and rectangle	• Can identify shapes in the given figure and give appropriate reason • Describes relationship between shapes of cuboids and the net of cuboids. • Gives reason for tiles of a given region using a given tile shape • Create patterns using with shapes.		• can connect the knowledge of 2D shapes to real life objects in their surroundings	• can draw 2D shapes on grid paper • can divide into two halves and represents halves in a whole • identify different shapes using different colours into different shapes
Day to Day maths	• Money • Length	• Can solves the problems with contextual day to day life situations		• can make new problems on daily situations	• can solve day to day life problems	• they can make bill

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Weight • Capacity • Time	involving four fundamental operations with measurements (money, length, weight, capacity, time) • Prepares rate charts and bills			involving more than two concepts / operations	
Measurements	• Length • Weight • Capacity • Time	• Can measure the lengths at objects by using scale and calculate the lengths • Can measure the capacity of a vessel by using small vessels can calculate its capacity (by using standard unit vessels also) • Can solve the problems on capacity and time • Can identify by the date, week, month and years through calendar.	• Estimates the length at given objects like tables blackboard etc in standard units (cm's) • Estimate weight and capacity in standard units • Can identify which is heavy and which is lighter objects • They can select the combination of small weight equal to the given weight • Can find the simple patterns of numbers from the calendar to foarm a square.	• Can explain the need of standard units in measuring lengths, capacity weight • Appreciates the conservations of weight and capacity	• Can use of the concepts at length, weight, capacity time in daily life problems	• Can represents the time in the clock (only hours)
Data handling	• Collection of data • Organizing data • Tally marks • Pictographs	• Collect the suitable data for the tabulating		• Comments on the data		• Represent the data in tabular form • Represent the data with tally marks.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : IV MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Geometry	• Shapes and special understandings	• Pupil can identify 3D shapes in objects • Can identify edges and corners of 3D shapes • Can identify the side view, top view, front view of 3D-objetcs • Can identify the nets of cuboid and cube shaped boxes • Can identify 2D-shapes in objects (3D) • Can understand the meaning perimeter and solve the problems regarding perimeter of 2D shapes	• Pupil can distinguishes among 3D-shapes based on their ability to roll and slide • Can give reasons for the sequences for patterns	• Pupils can describes the 3D shapes objects features. • Can explain about the pattern • Can give example for 3D – objects (i.e. cube, cuboid)		• Make picture using known 2D shapes • Can make shapes using dotted board • Pupil can explore line symmetry through reflective paper cutting and paper folding etc • Can draw the nets for cube and cuboid.
NUMBERS	• Numbers up to 1000	• Pupils can solve the problems related to 2-3 digit numbers using word problems • Can expand 2,3 digit numbers by using place values	• Can compare 2 and 3 digit numbers and give reasons • Can arrange the given numbers in ascending and descending order	• Can read and write 2,3 digit numbers		• Represents 2-3 digits numbers on number line • Represents the 2-3 digit numbers through objects and pictures (currency)

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• ADDITION & SUBTRACTION	• Can solve addition and subtraction problems up to 999 in different situations (through contextual situations, pictures word problems and numerals) • Can solve the addition and subtraction problems horizontally and vertically(in different methods)	• Can estimate sums and differences of 2,3 digit numbers and give reasons • Can verify the results of addition or subtraction of given numbers	• Can write a situation / problem regarding addition and subtraction in mathematical expression by using symbols. • Can frame new word problems sclated to addition and subtraction	• can solve the problems having both addition and subtractions	• can represent the addition and subtraction on number line.
	• Multiplication	• Can solve the problems related to multiplications of 1,2 digit numbers with 1 & 2 digit values • Can solve the multiplication problems in different methods (using standard algorithm, distributive law) • Can multiply the numbers 2 & 3 digit numbers by 10s and 100s	• Estimate the results of multiplication and give reasons	• Can write a situation / problem regarding multiplication in the mathematical expression by using symbols. • Can make word problems on multiplications	• Can solve multiplication problems involving different concepts and operations.	•
	• Divisions	• Can solve the problems on division of 2 & 3 digit numbers by 1 & 2 digit values (with reminder and without reminder)	• Can estimate the result of the division problems without doing the process, and verify it	• Can write the conceptual problems into mathematical form • Can make new problems on	• Can identify the relation between division and multiplication • Can solve the problems on	

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
				division (grouping & equal sharing) • Can explain the division by using the terms devisor, divided quotient & reminder	division involving concepts and different situations.	
	• Fractional numbers	• Can identify the half, one forth, and three forth of a hole. • Can identify other fractions such as 3/2, 5/2, 5/4 etc… • Can add and subtract like fractions (institutively)	• Can compare the fractions 1/2, 1/4 and 3/4 and give reasons.	• Can write fractional numbers half (1/2), one forth (1/4) and three forth (3/4) of a hole. • Explain the meaning of 1/2, 1/4, 3/4	• Can understand the relationship between division and fraction	• Can represent the fractions 1/4, 1/2, ¾ through pictures.
	• Patterns	• Can identify the sequence in the given number patterns and carry forward the pattern	• Find the sequence in the pattern and give proper reason. • Can verify the given pattern.	• Can create new patterns on their own.		
Day to day mathematics	• Involving in the daily life situations regarding money, length, capacity, weight and space.	• Can solve the problems of day to day life situations selected to money, length, capacity, weight and space etc.	• Can estimate the results of day to day life problems and give appropriate reasons.	• Can create word problems related to day to day life situations involving different concepts / situations.	• Solves day to day life problems using different methods and concepts (using more than 2 concepts / multiple stage of solving	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Measurement	• Length	• Can solve the problems related with length in different contextual situational problems in different methods. • Can measure the lengths of the objects in m and cms	• Can estimate length of the objects and distance between to given location from a point	• Can express the lengths of the objects in different units cm,m • Can convert the units of length meter to centimeter • Can create new problems / word problems on lengths	• Can solve the problems on lengths involving various concepts(+, -, ÷) in day to day life	
	• Weight	• Can solve the problems involving weight using kgs and grams.	• Can estimate weights of an object and verifies it using a balance • Can identify the relationship between kg and gram	• Can use and convert kgs to grams (vice versa) • Can appreciate the conservation of weights • Can create new problems on weight	• Can solve the problems on weight involving various concepts / operation (+, -, x, ÷) in day to day life.	
	• Capacity	• Can solve the problems involving capacity using liter and ml.	• Can estimate capacity of a container and verifies it by measuring • Can identify relationship between liter andml.	• Can use and convert liter and ml (vice versa) • Can appreciate the conservation of capacity • Can create new problems on capacity	• Can solve the problems on capacity involving various concepts / operations (+, -, x, ÷) in day to day life.	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Time	• Can solve the problems involving time. • Can select the date to the day on the calendar.	• Can understand the selection slip between hours and minutes. • Can identify correct time in the given clocks and gives reason. • Can distinguish between gen year and leaf year.	• Can appreciate the conservation of time. • Can create new problems on time.	• Can solve the problems on time involving various concepts / operations.	• Can represent the time in clock (hours – minutes).
Data Handling	• Collecting data and organizing data (using tally marks). • Reading bar graph and picto graph.	• Can organize the raw data into classified data. • Can solve the problem / interpretation of data and draw conclusions.	• Can analyse the data.	• Can explain inferences of the given data.	• Can use concept of picto graphs in daily life situations.	• Can represent the data in tally marks. • Can read and represent the data in tabular form. • Can read the bar graphs. • Can read the picto graphs.

'Problem Solving' means engaging in a task for which the solution method is not known in advance. In order to find a solution, one must draw on one's knowledge, and through this process, one develops new mathematical understanding. Solving problems is not only a goal of learning mathematics but also a major means of doing so. When one arrives at the correct solution there is naturally a great deal of satisfaction and sense of self-confidence which gets generated. And that, surely, is one of the things that any teacher is trying to inculcate in a student.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : V MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
1) Geometry	• Shapes and special understanding	• Can identify the nets the shapes at cubes, cuboids • Can draw the different views at simple objects plans • Can explore line of symmetry in familiar 3D objects expressed as 2D shapes. • Can explore the perimeter and area to simple shapes • Can understand angle through observation an paper folding. • Can identify and draw the right angle, more than and less than right angle.	• Can use shapes to create different shapes and different patterns by using tangrams. • Can explore rotation and reflections of familiar 2D shapes • Can estimates the area & perimeter of shapes	• Can identify and express the center and radius of a circle. • Can explain area & perimeter of 2D shapes.	• Can identifies the floor maps, roots / road maps by connecting the knowledge of 2D & 3D shapes.	• Can draw the shapes on dotted paper • Can draw a simple floor map of familiar locations by using indications (point, line, vertex, ray)
NUMBERS	• Number concept • Addition • Subtraction • Multiplication • Division	• Can expands the numbers using place values. • Can solve the word problem for addition and subtraction (upto 99999)	• Can compare the numbers using the contextual situation (up to 5 digits) • Can forms numbers using given digits	• Can frame the word problems involving four fundamental operations	• Can explore the relation between multiplication and division by using 2 and 3 digit numbers.	• Can represent the simple fractions on number line

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can multiply 3 digits x 2 digits using standard algorithm as well as the distributive law. • Can divide 2 digit by 2digt, 3 digits by 2 digits with remainder. • Can identify the even and add numbers • Can do addition and subtraction at like fractions.	• Can estimate the sums and differences at 3,4 digit numbers • Can answer the product at the number multiplying by 10's, 100's, and 1000's by oral and written • Estimates the product at 3 digit x 1 digit and 3 digit x 2 digit numbers. • Can say the divisible rules for 2,5 & 10 • Estimates the quotients		• Can apply simple fractions to measurements.	
Day to Day Maths	• Understanding and solving problems in daily life situations	• Can solve the problems related to daily life situations	• Can estimate the result / answer in daily life problems	• Can explain the method to solve problems in daily life situations	• Can solve word problems / contextual situations using more than one operations (or) more than one concept (or) multiple stages at solving	
Measurements	• Length • Weight • Capacity	• Can apply the four operations in solving problems involving length, weight and capacity	• Can estimate length, weight, capacity at a solid body.	• Can relates commonly used larger and smaller units of length, weight	• Can determine intuitively are and perimeter	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can understand the concept of area & can solve the problems		and capacity and converts one to the other. • Can convert fractional larger unit into complete smaller unit.	• Can apply simple fractions to quantities	
Time	• Time	• Can complete the number of days between two dates. • Can find time intervals in simple cases using addition and subtraction.		• Converts hours into minutes and seconds • Can express the time using the terms A.M and P.M • Can convert 12 hours time to 24 hours clocks (vise versa)		
Data Handling	• Reading Data and using picture graphics	• Can interpretation the data given in tables	• Can analyse the given data in tables			• Can understand the importance of appropriate scale for picto graph • Can read the data using bar graphs • Can organize the data using tally marks •

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Patterns	• Understanding simple patterns	• Can draw symmetric pictures and symmetric axis	• Can identify the patterns in square numbers and triangular numbers • Can identify the patterns in multiplication and division • Can make border strip and tailing patterns • Identifies the blacks are units at the pictures.			

Visualization and representation are skills that mathematics can help to develop. Modeling situations using quantities, shapes and forms are the best use of mathematics, mathematical concepts can be represented in multiple ways and these representation can serve a variety of purposes in different contexts. All of this adds to the power of mathematics. For example a function may be represented in algebraic form or in the form of a graph. The representation p/q can be used to denote a fraction as a part of the whole, but can also denote the quotient of two numbers, pand q. Learning this about fractions is as important, if not more, than learning the arithmetic of fractions.

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6. Mathematics Syllabus – Academic Standards – Elementary Secondary Level

a. Syllabus - Classes VI to X

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
Number System (60 hrs) **(i) Knowing our Numbers:** Consolidating the sense of Numberness up to 99,999 (five digits) • Estimation of numbers • Comparison of numbers • Place value (recapitulation and extension); • Connectives: use of symbols =, $<$, $>$. • Word problems on number operations involving large numbers up to a maximum of 5 digits in the answer (This would include conversions of units of length & mass from the larger to the smaller units) • Estimation of outcome of number operations. • Introduction to large numbers a) up to lakhs and ten lakhs b) up to crores and ten crores • Approximation of large numbers • International system of numbers (Millions.) • Use of Large numbers in daily life situations. **(ii) Whole numbers**	**Number System (50 hrs)** **(i) Integers** • Addition, Subtraction, Multiplication and Division of integers (through patterns). • Properties of integers in addition, multiplication & division through patterns (closure, commutative, associative, inverse and including identities including identities, and distributive property) (examples from whole numbers as well). ○ Expressing properties in a general form. ○ Construction of counter examples, (e.g. Subtraction is not commutative). ○ Multiplication and division by zero • Word problems involving integers (on all operations) **(ii) Fractions Decimals and rational numbers:** • Multiplication of fractions • Fraction as an operator "of" • Division of fractions • Reciprocal of a fraction and its use	**Number System (50 hrs) (67periods)** **(i) Playing with numbers** • Writing and understanding a 2 and 3 digit number in generalized form (100a + 10b) +c, where a, b, c can be only digits 0-9) and engaging with various puzzles concerning this. (Like finding the missing numerals represented by alphabets in sums involving any of the four operations) Children to solve and create problems and puzzles. • Number puzzles and games • Understanding the logic behind the divisibility tests of 2, 3,4, 5, 6, 7,8,9, and 11 for a two or three digit number expressed in the general form. • General rule of	**Number System (20 periods)** • **Real numbers** Review of representation of natural numbers, integers, and rational numbers on the number line. • Representation of terminating / non terminating recurring decimals, on the number line through successive magnification. • Rational numbers as recurring / terminating decimals. • Finding the square root of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ correct to 6-decimal places by division method • Examples of nonrecurring / non terminating decimals such as 1.01011011101111--- 1.12112111211112--- and $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc. • Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$, and their representation on the number line. • Existence of each real number on a number line by using Pythogorian result. • Concept of a Surd. • Rationalisation of a monomial, binomial surds of second order.	**Number System** **(i) Real numbers (15 periods) :** • More about rational and irrational numbers. • Fundamental Theorem of Arithmetic – statements. • Proofs of results, irrationality of $\sqrt{2}$, $\sqrt{3}$ etc. and decimal expansions of rational numbers in terms of terminating, non terminating, recurring of decimals and vice versa. • Properties of real numbers • Introduction of logarithms • Conversion of a number in exponential form to a logarithm tic form • Properties of logarithms $\log_a a = 1$; $\log_a 1 = 0$ • Laws of logarithms $\log xy = \log x + \log y$; $\log x/y = \log x - \log y$ $\log x^n = n \log x$ • Standard base of logarithms and usage

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
• Natural numbers, whole numbers • Properties of numbers (closure, commutative, associative, distributive, additive identity, multiplicative identity) • Division by zero • Number line- Binary operations (addition, subtraction, multiplication) on the number line • Seeing patterns, identifying and formulating rules to be done by children. • Utility of properties in fundamental operations **(iii) Playing with Numbers:** • Consolidating divisibility rules of 2,3,5,6,9,10 • Discovering divisibility rules of 4,8,11 through observing patterns. • Multiples and factors, • Prime & composite numbers, Co-prime numbers and twin prime numbers, • Prime factorization, every number can be written as products of prime factors. • HCF and LCM, prime factorization and division method. • Property LCM × HCF = product of two numbers. • LCM & HCF of co-primes.	• Word problems involving mixed fractions (related to daily life) • Introduction to rational numbers • Multiplication and division of decimal fractions • Conversion of units (length & mass) • Comparison of rational numbers.	divisibility by any number. **(ii) Rational Number** • Properties of rational numbers. (including identities). • Using general form of expression to describe properties. Appreciation of properties. • Representation of rational numbers on the number line • Between any two rational numbers there lies another rational number • Representation of rational numbers as decimal (denominators other than 10, 100,….) • Representation of decimal numbers (terminating, non terminating but recurring) in rational form. • Consolidation of operations on rational numbers. • Word problems on rational numbers (all operations) • Word problem (higher logic, all operations, including		**(ii) Sets (8 periods):** • Sets and their representations : Empty set, Finite and infinite sets. Equal sets. Subsets, subsets of the set of real numbers (especially intervals with notations). Universal set and cardinality of sets. • Venn diagrams : Union and intersection of sets. Difference of sets. Complement of a set. Disjoint sets.

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
(iv) Negative Numbers and Integers • How negative numbers arise, models of negative numbers, connection to daily life, ordering of negative numbers, representation of negative numbers on number line. • Understanding the definition of integers, identification of integers on the number line • Comparison of integers, ordering of integers by using symbols • Operation of addition and subtraction of integers, showing the operations on the number line (Understanding that the addition of negative integer reduces the value of the number) **(V) Fractions and Decimals:** • Revision of what a fraction is, Fraction as a part of whole • Representation of fractions (pictorially and on number line) • Fraction as a division, proper, improper & mixed fractions, equivalent fractions, like, unlike fractions. • Comparison of fractions • Addition and subtraction of		ideas like area) **(iii) Square numbers, cube numbers, Square roots, Cubes, Cube roots.** • Square numbers and square roots. • Square roots using factor method and division method for numbers containing. a)not more than 4 digits and b)not more than 2 decimal places • Pythagorean triplets and problems involving Pythagorean triplets. • Cube numbers and cube roots (only factor method for numbers containing at most 3 digits). • Estimating square roots and cube roots. Learning the process of moving nearer to the required number.		

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
fractions • Word problems (Avoid large and complicated calculations) • Review of the idea of a decimal fraction • Place value in the context of decimal fraction, inter conversion of fractions and decimal fractions (avoid recurring decimals at this stage) • Word problems involving addition and subtraction of decimals (word problems should involve two operations) • Contexts: money, mass, length.				
Algebra (15 hrs) **(i) Introduction to Algebra** • Introduction to variable through patterns and through appropriate word problems and generalizations (example $5 \times 1 = 5$ etc.) • Generate such patterns with more examples. • Introduction to unknowns through examples with simple contexts (single operations) • Rules from Geometry and Menstruation. **(ii)Simple Equations** • Introduction • Solution of simple equation	**Algebra (20 hrs)** **(i)Exponents and powers** • Meaning of x in a^x where $a \in Z$ • Writing a number in the exponential form through prime factorization. • Laws of exponents (through observing patterns to arrive at 5 generalizations) where $m, n \in N$ (i) $a^m a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m/a^n = a^{m-n}$, where $(m, n) \in N$ (iv) $a^m.b^m = (ab)^m$ (v) number with exponent zero	**Algebra (20 hrs)** **(27 periods)** **Exponents & powers** **i) Powers** • Decimal numbers in exp. notation. • Integers as exponents. • Laws of exponents with integral powers • Representing large numbers in standard (scientific) notation. **ii) Algebraic Expressions** • Addition and subtraction of algebraic expressions • Multiplication and division of algebraic	**Algebra** **(i)Polynomials (25 periods)** • Definition of a polynomial in one variable, its coefficients, with examples and counter examples, its terms, zero polynomial. • Constant, linear, quadratic, cubic polynomials; monomials, binomials, trinomials. Zero / roots of a polynomial / equation. • Division of polynomials • State and motivate the Remainder Theorem with examples and analogy to integers (motivate). • Statement and verification of the Factor Theorem. • Recall of algebraic expressions and identities. • Further identities of the type:	**Algebra** **(i) Polynomials (8 periods)** • Zeros of a polynomial. • Geometrical meaning of zeros of quadratic and cubic polynomials using graphs. • Relationship between zeros and coefficients of a polynomial with particular reference to quadratic polynomials. • Statement and simple problems on division algorithm for polynomials with integral coefficients. **(ii) Pair of Linear Equations in Two Variables (15 periods)** • Pair of linear equations in two variables. Geometric representation of different

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
by Trial and Error method.	• Terms with negative base. • Expressing large number in standard form (Scientific Notation) **(ii)Algebraic Expressions Introduction** Generate algebraic expressions (simple) involving one or two variables • Identifying constants, coefficient, powers • Like and unlike terms, degree of expressions e.g., x^2y etc. (exponent ≤ 3, number of variables ≤ 2) • Types of algebraic expressions. • Addition, subtraction of algebraic expressions (coefficients should be integers). • Finding the value of the expression. **(iii)Simple equations** • Simple linear equations in one variable (in contextual problems) with two operations (integers as coefficients)	exp.(Coefficient should be integers) • Identities: Derivation and geometric verification of $(a \pm b)^2 = a^2 \pm 2ab + b^2$, $a^2 - b^2 = (a - b)(a + b)$ • Factorization (simple cases only) as examples of the following types $a(x + y)$, $(x \pm y)^2$, $x^2 - y^2$, $(x + a).(x + b)$ **(iii)Simple equations** • Solving linear equations in one variable in contextual problems involving multiplication and division (word problems) (with integral coefficient in the equations)	$(x+y+z)^2 = x^2+y^2+x^2+2xy+2yz+2zx$ $(x\pm y)^3 = x^3\pm y^3\pm 3xy\ (x\pm y)$ $x^3 +y^3 +z^3 -3xyz = (x+y+z)$ $(x^2 +y^2 +z^2 -xy-yz-zx)$ $x^3 +y^3 = (x+y)(x^2 - xy + y^2)$ $x^3 -y^3 = (x-y)(x^2 + xy + y^2)$ and their use in factorization of polynomials. Simple expressions reducible to these polynomials. **(ii)Linear Equations in Two Variables (12 periods)** • Recall of linear equations in one variable. • Introduction to the equation in two variables. • Solution of a linear equation in two variables • Graph of a linear equation • Equations of lines parallel to x-axis and y-axis. • Word problems related to linear equations	possibilities of solutions / inconsistency. • Algebraic conditions for number of solutions. • Solution of pair of linear equations in two variables algebraically – by substitution, by elimination and by cross multiplication. Simple situational problems. • Simple problems on equations reducible to linear equations. **(iii) Quadratic Equations (12 periods)** • Standard form of a quadratic equation $ax^2+bx+c=0$, $(a \neq 0)$. • Solutions of quadratic equations (only real roots) by factorization and by completing the square, i.e., by using quadratic formula. • Relationship between discriminant and nature of roots. • Problems related to day-to-day activities. **(iv) Progressions (11 periods)** • Sequence and series • Motivation for studying AP. Derivation of standard results of finding the n^{th} term and sum of first n terms. • Motivation for studying G.P • n^{th} term of G.P.

<table>
<tr><th>Class – VI</th><th>Class – VII</th><th>Class – VIII</th><th>Class – IX</th><th>Class – X</th></tr>
<tr>
<td>Ratio and Proportion(15hrs)
• Concept of Ratio
• Ratio in different situations.
• Comparison of ratios of different units
• Division of a quantity in a given ratio.
• Proportion as equality of two ratios
• Unitary method (with only direct variation implied)
• Word problems
• Understanding ratio and proportion in Arithmetic.</td>
<td>Ratio - Applications (20 hrs)
• Ratio and proportion (revision)
• Unitary method continued, consolidation, general expression.
• Direct proportion
• Percentage- an introduction.
• Understanding percentage as a fraction with denominator 100.
• Converting fractions and decimals into percentage and vice-versa.
• Application to profit and loss (single transaction only)
• Discount.
• Application to simple interest (time period in complete years).</td>
<td>Business Mathematics (25 hrs)
• Compound ratio – Word problems.
• Problems involving applications on percentages, profit & loss, overhead expenses, Discount, tax. (Multiple transactions)
• Difference between simple and compound interest (compounded yearly up to 3 years or half-yearly up to 3 steps only), Arriving at the formula for compound interest through patterns and using it for simple problems.
• Direct variation – Simple and direct word problems. Inverse variation – Simple and direct word problems. Mixed problems on direct , inverse variation
• Time & work problems– Simple and direct word problems
• Time & distance : Simple and direct word problems</td>
<td></td>
<td>Trigonometry
(i) Introduction (15 periods)
• Trigonometric ratios of an acute angle of a right-angled triangle i.e. sine, cosine, tangent, cosecant, cotangent.
• Motivate the ratios, whichever are defined at 0^0 and 90^0.
• Values (with proofs) of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.
• Trigonometric Identities: Proof and applications of the identity $\sin^2A+\cos^2A=1$. $1+\tan^2A=\sec^2A$ $\cot^2+1=\mathrm{cosec}^2A$ only simple identities given.
• Trigonometric ratios of complementary angles.
(ii) Applications of trigonometry (8 periods)
• Angle of elevation, angle of depression
• Simple and daily life problems on heights and distances. Problems should not involve more than two right triangles and angles elevation/ depression should be only 30^0, 45^0, 60^0.</td>
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Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
			Coordinate geometry (9 periods) • Introduction • Cartesian system • Plotting a point in a plane if its co-ordinates are given.	**Coordinate geometry Lines (In two-dimensions) (15 periods)** • Review the concepts of coordinate geometry done by the graphs of linear equations. • Distance between two points i.e. p (x_1, y_1) and q (x_2, y_2) • Section formula internal division of a line segment in the m:n. • Area of a triangle on coordinate axis. • Slope of a line joining two points.
Geometry (65 hrs) **i) Basic geometrical ideas (2-D):** • Introduction to geometry. Its linkage with and reflection in everyday experience. • Point, Line, line segment, ray. • Open and closed figures. • Curvilinear and linear boundaries • Interior and exterior of closed figures. • Angle — Vertex, arm, interior and exterior, Triangle — vertices, sides, angles, interior and exterior. • Quadrilateral — Sides, vertices, angles, diagonals, adjacent sides and opposite	**Geometry (60 hrs)** **(i) Lines and Angles** • Pairs of angles (linear pair) 1. complementary, 2. supplementary, 3. adjacent, vertically opposite angles. (verification and simple proof of vertically opposite angles) • Transversal – Angles formed by the transversal. • Properties of parallel lines with transversal (alternate, corresponding, interior, exterior angles, interior angles on the same side of transversal. **(ii) Triangles:** • Definition of triangle. • Types of triangles acc. to sides and angles	**Geometry (40 hrs)** **(i) Construction of Quadrilaterals: (54 periods)** • Review of quadrilaterals and their properties. • Four sides, one angle • Four sides, one diagonal • Two adjacent sides, three angles • Three sides, two diagonals. • Three sides, two angles in between. **(ii) Representing 3-D in 2-D** • Identify and Match pictures with objects [more complicated	**Geometry** **(i) Introduction to Euclid's Geometry (6 periods)** • History – Euclid and geometry in India. Euclid's method of formalizing observed phenomenon onto rigorous mathematics with definitions, common / obvious notions, axioms / postulates, and theorems. The five postulates of Euclid. Equivalent **varies** of the fifth postulate. Showing the relationship between axiom and theorem. • Given two distinct points, there exists one and only one line through them. • (Prove) Two distinct lines cannot have more than one point in common. **(ii) Lines and Angles**	**Geometry** **(i) Similar triangles (18 periods)** Definitions, examples, properties of similar triangles. • Difference between congruency and similarity of triangles. • (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. • (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side. • (Motivate) If in two triangles, the corresponding

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
sides, adjacent and opposite angles (only convex quadrilateral are to be discussed), interior and exterior of a quadrilateral. • Circle — Centre, radius, diameter, chord, arc, sector, segment, semicircle, circumference, interior and exterior. **(ii) Measures of Lines and Angles:** • Measure of Line segment • Types of angles- acute, obtuse, right, straight, reflex, complete and zero angle. • Measure of angles • Pair of lines Intersecting and perpendicular lines and parallel lines **iii) Practical Geometry (Constructions)** • Drawing of a line segment (using Straight edged Scale, compasses) • Construction of circle • Perpendicular bisector • Drawing a line perpendicular to a given line from a point a)on the line b)outside the line. • Construction of angles (using protractor)	• Properties of triangles • Sum of the sides, difference of two sides. • Angle sum property (with notion of proof and verification through paper folding, proofs , using property of parallel lines , difference between proof and verification • Exterior angle property of triangle • Median and Altitude of a triangle. **(iii) Congruence**: • Congruence through superposition **ex**. Blades, stamps etc.. • Extend congruence to simple geometrical shapes ex: Triangle , Circles, • Criteria of congruence (by verification only) • Property of congruencies of triangles SAS, SSS, ASA, RHS Properties with figures **(iv) Construction of triangles (all models)** • Constructing a Triangles when the lengths of its 3 sides are known (SSS Criterion) • Constructing a triangle when the lengths of 2 sides and the measures of the angles between them are known (SAS criterion)	e.g. nested, joint 2-D and 3-D shapes (not more than 2)]. • Drawing 2-D representation of 3-D objects (Continued and extended) with isometric sketches. • Counting vertices, edges & faces & verifying Euler's relation for 3-D figures with flat faces (cubes, cuboids, tetrahedrons, prisms and pyramids) **(iii)Exploring geometrical figures** • Congruent figures • Similar figures • Symmetry in geometrical figures w.r.t. to triangles, quadrilaterals and cirlcles. Revision of reflection symmetry, rotational symmetry and applications • Point symmetry • Estimation of heights and distances by similar figures • Dilations • Tessellations •	**(10 periods)** • (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180^0 and the converse. • (Prove) If two lines intersect, the vertically opposite angles are equal. • (Motivate) Relation between corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines. • (Motivate) Lines, which are parallel to given line, are parallel. • (Prove) The sum of the angles of a triangle is 180^0. • (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. **(iii) Triangles (20 periods)** • (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle (SAS Congruence). • (Prove) Two triangles are congruent if any two angles and the included side of one triangle are equal to any two angles and the included side of the other triangle (ASA Congruence). • (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS	angles are equal, their corresponding sides are proportional and the triangles are similar (AAA). • (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar (SSS). • (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar. • (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides. • (Motivate) If a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other. • (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. • (Prove) In a triangle, if the square on one side is equal

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
• Angle equal to a given angle (using compass) • Angle 60°, 120° (Using Compasses) • Angle bisector- making angles of 30°, 45°, 90° etc. (using compasses) **vi) Understanding 3D, 2D shapes** • Identification of 3-D shapes: Cubes, Cuboids, cylinder, sphere, cone, prism (triangular), pyramid (triangular and square) Identification and locating in the surroundings • Elements of 3-D figures. (Faces, Edges and vertices) • Polygons- introduction, types of polygons, regular polygons **v) Symmetry: (reflection)** • Observation and identification of 2-D symmetrical objects for reflection symmetry • Operation of reflection (taking mirror images) of simple 2-D objects • Recognizing reflection symmetry (identifying axes) • Multiple lines of symmetry.	• Constructing triangle when the measures of 2 of its angles and length of the side included between them is given (ASA criterion) • Constructing a right angle triangle when the length of one leg hypotenuse are given (RHS criterion). • Constructing a triangle when the lengths of 2 sides and the measures of the non included angle are known (SSA criterion) **(v) Quadrilaterals** • Quadrilateral-definition. • Quadrilateral, sides, angles, diagonals. • Interior, exterior of quadrilateral • Convex, concave quadrilateral differences with diagrams • Angle sum property (By verification) , problems • Types of quadrilaterals • Properties of parallelogram, trapezium, rhombus, rectangle, square and kite. **(vi) Symmetry** • Recalling reflection, line symmetry, lines of symmetry for regular polygons. • Idea of rotational symmetry, observations of rotational symmetry of 2-D objects.		Congruence). • (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal to the hypotenuse and a side of the other triangle. • (Prove) The angles opposite to equal sides of a triangle are equal. • (Motivate) The sides opposite to equal angles of a triangle are equal. • (Motivate) Triangle inequalities and relation between 'angle and facing side'; inequalities in a triangle. **(iv)Quadrilaterals (10 periods)** • (Prove) The diagonal divides a parallelogram into two congruent triangles. • (Motivate) In a parallelogram opposite sides are equal and its converse. • (Motivate) In a parallelogram opposite angles are equal and its converse. • (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal. • (Motivate) In a parallelogram, the diagonals bisect each other and its converse. • (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and its converse. **(v)Area (4 periods)** • Review concept of area, recall area of a rectangle. • (Prove) Parallelograms on the same	to sum of the squares on the other two sides, the angles opposite to the first side is a right triangle. **(ii) Construction:** • Division of a line segment using basic proportionality theorem. • A triangle similar to given triangle as per the given scale factor. **(iii) Tangents and secants to a circle (15 periods)** • Tangents to a circle motivated by chords drawn from points coming closer and closer to the point. • (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact. • (Prove) The lengths of tangents drawn from an external point to a circle are equal. • Segment of a circle made by the secant. • Finding the area of the minor/ major segment of a circle. **(iv) Constructions** A tangent to a circle through point given on it. • Tangent to a circle from a point outside it.

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
	(90°, 120°, 180°) • Operation of rotation through 90° and 180° of simple figures. • Order of rotational symmetry • Examples of figures with both rotation and reflection symmetry (both operations) • Examples of figures that have reflection and rotation symmetry and vice-versa **(vii) Understanding 3-D in 2-D shapes:** • Nets for cube, cuboids, cylinders, cones and tetrahedrons. • Drawing 3-D figures in 2-D showing hidden faces through oblique sketches and Isometric sketches.		base and between the same parallels have the same area. • (Motivate) Triangles on the same base and between the same parallels are equal in area and its converse. **(vi)Circles (15 periods)** • Definitions of circle related concepts of circle; radius, circumference, diameter, chord, arc, subtended angle. • (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse. • (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and its converse • (Motivate) There is one and only one circle passing through three non-collinear points. • (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre (s) and its converse. • (Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. • (Motivate) Angles in the same segment of a circle are equal. • (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line segment, the four points lie on a circle. • (Motivate) The sum of the either pairs of the opposite angles of a cyclic quadrilateral is 180^0 and its converse.	

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
			(vii)Constructions **(10 periods)** • Construction of a triangle given its base, sum / difference of the other two sides and one base angles. • Construction of a triangle when its perimeter and base angles are given. • Construct of segment of a circle containing given chord and angle.	
Mensuration (15 hrs) **Perimeter and Area** • Introduction and general understanding of perimeter using many shapes. • Shapes of different kinds with the same perimeter. • Perimeter of a rectangle – and its special case – a square. • Perimeter of regular polygons • Deducing the formula of the perimeter for a rectangle and then a square through pattern and generalization. • Concept of area, Area of a rectangle and a square. Counter examples to different misconcepts related to perimeter and area. • Word problems on perimeter and area.	**Mensuration (15 hrs)** **Area and Perimeter** • Revision of perimeter and Area of Rectangle, Square. • Area of parallelogram. • Area of a triangle • Area of rhombus. • Idea of Circumference of Circle. • Area of rectangular paths.	**Mensuration** **(15 hrs)** • Area of a triangle: formulae (without proof) and its application in finding the area of a quadrilateral. • Area of a trapezium • Area of the quadrilateral and other polygons. • Area of the circle & circular paths. • Surface area of a cube, cuboid • Concept of volume, measurement of volume using a basic unit, volume of a cube, cuboid • Volume and capacity.	**Mensuration (15 hrs)** **Surface Areas and Volumes (14 periods)** **Areas of Plane figures** **(4 periods)** • Revision of surface area and volume of cube, cuboid • Surface areas of right circular cylinder, cone, sphere, hemi sphere. • Volume of right circular cylinder, cone, sphere and hemi sphere • Word problems on cylinder, cone, sphere, hemi sphere.	**Mensuration** **I. Surface Areas and Volumes (10 periods)** • Problems on finding surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders / cones. • Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken.)

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
Data handling (10 hrs) • What is data • Collection and organisation of data - examples of organizing it in tally marks and a table. • Pictograph- Need for scaling in pictographs interpretation & construction. • Bar graphs: Interpreting bar graphs, drawing vertical and horizontal bar graphs for given data.	**Data handling (15 hrs)** • Collection and organisation of data. • Mean median and mode of ungrouped data – understanding what they represent. • Reading bar-graphs Constructing double bar graphs. • Simple pie charts with reasonable data numbers	**Data handling (15 hrs)** • Revision of Mean, Median and Mode of ungrouped data. • Determination of mean by Deviation Method. • Scope and necessity of grouped data. • Preparation of frequency distribution table • Cumulative frequency distribution table • Frequency graphs (histogram for equal and unequal class intervals, frequency polygon, frequency curve, cumulative frequency curves)	**Data handling (15 hrs)** **Statistics (13 periods)** • Frequency distribution for ungrouped and grouped data • Mean, Median and Mode of ungrouped frequency distributions (weighted scores). **Probability (12 periods)** • Feel of probability using data through experiments. Notion of chance in events like tossing coins, dice etc. • Tabulating and counting occurrences of 1 through 6 in a number of throws. • Comparing the observation with that for a coin. Observing strings of throws, notion of randomness. • Consolidating and generalizing the notion of chance in events like tossing coins / dice. • Relating probability to chances in life-events. • Visual representation of frequency outcomes of repeated throws of the same kind of coins or dice. • Throwing a large number of identical dice/coins together and aggregating the result of the throws to get large number of individual events. • Observing and aggregating numbers over a large number of repeated events. Observing strings of throws notion of randomness	**Data handling (15 hrs)** **(i) Statistics (15 periods)** • Revision of Mean, median and mode of ungrouped (frequency distribution) data. • Understanding, the concept of Arithmetic Mean, Median and Mode for grouped (classified) data. • The meaning and purpose of arithmetic Mean, Median and Mode • Simple problems on finding Mean, Median and Mode for grouped / ungrouped data. • Usage and different values and central tendencies through Ogives. **(ii) Probability (10 periods)** • Concept and definition of Probability. • Simple problems (day to day life situation) on single events simple using set notation. • Concept of complimentary events.

Class – VI	Class – VII	Class – VIII	Class – IX	Class – X
			Proofs in Mathematics • Mathematical Statement, Verification of statement • Mathematical Reasoning, Deductive reasoning • Theorems, Conjectures and Axioms • What is a Mathematical proof? Steps of Mathematical proofs.	**Mathematical Modeling (8 periods)** • Concept of Mathematical modeling • Discussing the broad stages of modeling – real life, situations (ration, proportion, percentage, probability, fair installments, payments etc.)

There is also need to make connections between mathematics and other subjects of study. When children learn to draw graphs, they should also be encouraged to think of Functional relationships in the sciences, including geology. Our children need to appreciate the fact that Mathematics is an effective instrument in the study of science.

- NCF 2005

b. Maths Academic Standards – Learning Indicators – Secondary Level

Academic Standards: Academic standards are clear statements about what students must know and be able to do.
The following are categories on the basis of which we lay down academic standards.

Content	Problem Solving	Reasoning Proof	Communication	Connections	Visualization & Representation
• Content areas of Maths	Using concepts and procedures to solve. ***a. Kinds of problems:*** Problems can take various forms- puzzles, word problems, pictorial problems, procedural problems, reading data, tables, graphs etc. **Problem Solving** • Reads problems • Identifies all pieces of information/data • Separates relevant pieces of information • Understanding what concept is involved • Recalling of (synthesis of) concerned procedures, formulae etc. • Selection of procedure • Solving the problem • Verification of answers of raiders, problem based theorems. ***b. Complexity:*** The complexity of a problem is dependent on • Making connections(as defined in the connections section) • Number of steps • Number of operations • Context unraveling • Nature of procedures	• Reasoning between various steps (involved invariably conjuncture). • Understanding and making mathematical generalizations and conjectures • Understands and justifies procedures • Examining logical arguments. • Understanding the notion of proof • Uses inductive and deductive logic • Testing mathematical conjectures	• Writing and reading, expressing mathematical notations (verbal and symbolic forms) Ex: • $3+4=7$ • $3\neq 5$ • $n_1+n_2= n_2+n_1$ • Sum of angles in a triangle is 180^0 • Creating mathematical expressions • Explaining mathematical ideas in her own words like- a square is closed figure having four equal sides and all equal angles • Explaining mathematical procedures like adding two or more digit numbers involves first adding the digits in the units place and then adding the digits at the tens place/ keeping in mind carry over. • Explaining mathematical logic	• Connecting concepts within a mathematical domain- for example relating adding to multiplication, parts of a whole to a ratio, to division. Patterns and symmetry, measurements and space • Making connections with daily life • Connecting mathematics to different subjects • Connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space • Connecting concepts to multiple procedures	• Interprets and reads data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures • Making tables, number line, pictograph, bar graph, pictures. • Mathematical symbols and figures.

c) CLASS WISE ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : VI MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Number system	• Knowing numbers	• Pupil can solves word problems on number operations involving large numbers upto a maximum of 5 digits in the answer	• Pupils can estimate of outcome of number operations • Can compare numbers upto large numbers with concept of place value • Can form different numbers by using given numbers and select biggest, smallest among them	• Pupil can write any five digit numbers in words and vice versa • Can show the comparison of five digit numbers using the symbols <, >, =.	• Pupil can understand the usage of large numbers in daily life (i.e. village population, income from land etc.)	• Pupil can express the numbers into expanded and compact form by using unit, ten, hundred, thousand blocks represent numbers through them
	• Whole numbers		• Can verify of properties of whole numbers such as closure, associative, inverse, identity, distributive, commutative(+,-, x)	• Can understand the need of whole number instead of natural numbers	• Can find the usage of whole numbers from their daily life • Can understand the relation between N and W	• Represents the whole numbers on the number line
	• Playing with numbers	• Simplification of numerical involving two a more brackets • Can test the divisibility rules	• Can find the logic behind the divisibility rules • Can understand relationship between	• Can use brackets involving fundamental operations	• Can establishes the relation among factors • Can	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can understand the use of LCM and HCF in different situations and find them in division, prime factorisation method	LCM and HCF of two numbers by verification, why this relation hold only in two numbers and see the pattern conclude		understand the use of LCM and HCF from their real life situations. • Finds the pattern in division multiplication tables.	
	• Integers	• Can solve the problems on additions, subtraction involving integers	• Can compare integers and ordering of integers • Difference of +, - between N, Z	• Can understand the necessity of set of integers	• Can find the connection among N,W and Z	• Can represent the integers on the number line • Can show the addition, subtraction on the number line.
	• Fractions and decimals	• Can adds subtracts, multiples like and unlike fractions (avoid complicated, large tasks) • Inter conversion of fraction and decimal fractions • Word problems involving +, - of decimals (two operations together on money, mass, length, temperature)			• Can connect between fraction, decimal fractions, decimal numbers	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Algebra	• Introduction of algebra	• Can find the value of the expression when substitutions a value in place of variable (simple expression can be taken and single operation)	• Can generalize the given patterns and express as algebra expression	• Can convert the ral life simple contexts into algebraic expression (vice versa)	• Can find the usage of algebraic expression when occurring the unknown values. • Can interlink the number system with algebraic system by usage of simple contexts.	• Can represent the even, odd number in general form as 2n, 2n+1
Arithmetic	• Ratio and proportion	• Can calculate compound, inverse ratio of rations. • Can solve word problems involving unitary method.	• Can compare the given ratios. • Can verify the rule of proportion involving the ratios. • Can give the reason why the same units can be taken in expressing of ratios	• Can write ratios in symbolic and equivalent fractional form	• Can observe the relation between line and work time and distance writing reading to proportions.	
Geometry	• Basic geometrical ideas		• Can differentiate the basic geometric shapes (triangle, circle, quadrilaterals)	• Can give the example of basic geometry shapes (from surface of the surrounding objects)	• Can visualize the basic geometric shapes from surroundings.	• Can give pictorial representation of basic geometric shapes.

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
			• Can differentiate and compare the quadrilaterals and triangle.		• Can understand the inter relation between various components of a circle (circle, semicircle, sector, diameter, radius, chord etc)	
	• Measures of lines and angles	• Can measure the given line segment	• Can compare the length of line segments by estimation and verification • Can classify the given angles • Can differentiate the pair of line as intersecting, perpendicular lines. • Can estimates the type of given angle • Can compare the given angles • Can round of an angle to nearest measure by estimation		• Can find the usage of elementary shapes and their measurements in surroundings	• Can draw a line segment with given measurement. • Can draw the given angle using apparatus.
	• Symmetry	• Pupil can find the symmetry of 2D shapes	• Explain differences between symmetry, non-symmetry pictures	• Explain the reflection symmetry and	• Observes and Identifies reflection	• Can drawn the axis of symmetry of

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
				their axes of symmetry of 2D shapes	symmetry in the surroundings	20 shapes
	• Constructions		• Can estimate the pair of given lines one perpendicular or not? • Can estimate given line is angular bisector or not			• Can drawn construct the line segment circle, perpendicular bisector, angle and angle bisetor
	• Understanding 2D, 3D shapes		• Can explain the differences of polygons and regular polygon and give reasons for given polygon is regular or not • Categories 3D shapes based on Faces, edges and verities (cube, cuboide, cylinder, sphere, cone, trysm and pyramid		• Observes the objects which have faces any regular polygon shapes in the surroundings • Can understand the relations between 3D objects and thin pictures	• Can select the objects to draw the regular polygons and give reasons for selecting it • Can represent shapes of 3D figures as 2D on the paper
Mensuration	• Perimeter and area	• Can solve the problems involving perimeter and area of rectangle and square. • Can solve word problems of perimeter and area	• Can differentiate perimeter and area of a given figure • Can identify the same perimeter different shapes from given shapes	• Can express the perimeter and area of a rectangle / square in formula and in words also.	• Can establish the relation between units to area and perimeter • Can solves the problems on perimeter and	• Can show the area of the polygon by shading the region.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
			• Can find the errors in solving problems of perimeter, area and rectify them.		area involving more various concepts.	
Data Handling	• Collecting data • Organising data • Bar graphs, and picto graphs	• Can organize the raw data in to classifies data • Can solve the problems / interpretation of the data and draw conclusions		• Can explain merits and demerits of bar graphs and picto graphs • Can explain inferences of the given data	• Can understand the usage of bar graphs, pictographs in daily life situations. (year wise population, annual budget, production of crops etc)	• Can represent the data in tally marks • Can represent the data in tabular form • Can represent the data into bar graphs and pictographs.

The importance of systematic reasoning in mathematics cannot be over emphasized and is intimately tied to notions of aesthetics and elegance so dear to mathematicians. Proof is important, but in addition to deductive proof, children should also learn when pictures and constructions provide proof. Proof is a process that convinces a skeptical adversary; school mathematics should encourage proof as a systematic way of argumentation. The aim should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : VII MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
1) Number System.	• Integers • Fractions, decimals & Rational number	• Pupil can solve the problems involving four fundamental operations of integers. • Can solve the word problems involving the integers • Can use brackets for solving problems to simplify numerical statements. • Pupils can solve the problems in all operations of fractions • Can solve the word problems of all operations of decimal fractions • Can convert the small units into large units and vice versa.	• Pupils can explain why the division by zero is meaning less • Can differentiates and compares the set of natural numbers with integers • Can differentiates rational numbers with fractions • Can justify density property in rational numbers • Can verify properties of integers such as closure, associate inverse, identify, distributive, commutative (+, -, x)	• Can express the number properties of integers in general form. • Can use negative symbol in different contexts • Can express the need of set of rational numbers • Can express the properties of rational number in general form.	• Can find the usage of integer from their daily life situations. • Can understand the relation among N, W and Z. • Can find the usage of inter relation among fractional, rational numbers and decimal numbers	• Pupil can represent the integers on number line. • Can perform the operations of integers on the number line. • Can represent rational numbers on the number line. • Can represent the rational numbers in decimal form.
ALGEBRA	• Exponents and powers	• Pupils can write the large numbers in	• Can generalize the exponential laws	• Can understand the meaning of	• Can use prime factorization in	• Can express the large numbers

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
	• Algebraic expressions • Simple equations	exponential form by using prime factorization. • Can find the degree of algebraic expressions • Can do additions subtractions of algebraic expressions (co-efficient should be integers) • Can solve the word problems involving two operations (which can be expressed as simple equation and single variable)	through the observation of patterns. • Can generalize and generate algebraic expressions involving one or two variables by using the patterns.	'x' in ax where a & z • Can know the use of exponential form when using large numbers. • Can write the standard form of first, second, third order expressions in one or two variables. • Can converts the daily life problems into simple equations (contains one variable only)	expression of large numbers in exponential form. • Can use closure, commutative etc, properties in addition and subtraction of algebraic expressions. • Can use the simple equations involving problems related to daily life.	in standard form. • Can represent the algebraic expressions in standard form.
ARITHEMETIC	• Ratio applications	• Pupils can find the compound, inverse ratio of a ratio. • Can solve word problems involving unitary method • Can solve the word problems involving percentage concept. • Can solve the word problems to find simple interest	• Can compare the decimals, converting into percentage and vice versa. • Can formulate the general principles of ratio and proportions.	• Can express the fractions into percentages and decimal forms and their usage.	• Can use profit and loss concepts in daily life situations (single transactions only) • Can understand and use the	• Can convert fractions and decimals into percentage from and vice versa.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		(Time period in complete years)			solution for percentage problems in daily life	
Understanding shapes / geometry	• Lines and angles	• Can solve problems on angles made by transversal intersecting parallel line.	• Can differentiate the types of pairs of angle from given angles. • Can verify the parallel ness of the given lines with the use of properties of parallel lines. • Can prove and verify the angle sum property through paper folding and using property of parallel lins	• Can give examples for pair of angles	• Can observe the parallel ness in surroundings	• Can represent the notation of angle.
	• Triangle and its properties	• Can determine whether the given lengths of sides of shapes suitable to make triangle. • Can find the angle which is not given from exterior and other angles of triangle.	• Can make relationship between exterior angle to its opposite • Can classify the given triangle on the basis of sides and angles. • Can estimate the kinds of triangle by observing the given triangle.	• Can explains the different types of triangle according to sides and angles. • Can explain the property of exterior angle of triangle.	• Can use the concept of triangle in daily life.	
	• Congruency of triangles	• Pupils can identify the congruent triangles from given triangles suitable to make triangle.		• Can appreciate the congruency in 2D figures.		• Can represent the congruent triangles using symbols, notation.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Construction of triangles.	• Can construct triangles, according to given measurements.				
	• Quadrilateral		• Can differentiate the convex, concave quadrilaterals • Can verify and justify the sum angle property of quadrilateral	• Can explain the inter relationship between triangle and quadrilateral. • Can explain the different types quadrilaterals based on their properties.	• Can classify the given quadrilaterals using their properties and their inter relationship.	
	• Symmetry	• Can rotate the figure and find its angular symmetry	• Can differentiate linear and reflection symmetry using objects of figures.	• Can give examples that have reflection symmetry		
	• Understanding 3-D and 2-D shapes	• Can identify and count the no. of faces, edges, vertices and nets for 3-D fig. (Cube, cuboid, cone, cylinder)	• Can match picture with 3-d objects			• Can draw simple 3-d shapes into 2-d figure.
Mensuration	• Area and perimeter	• Can solve the problems of area, and perimeter for square, rectangle, parallelogram, triangle and rhombus shape of things.	• Can understand the relationship between square, rectangle, parallelogram, rhombus with triangle shapes (for finding the area of triangle)	• Can explain the concept of measurement using a basic unit.	• Applies the concept of area and perimeter to find / solve daily life situations problems (square,	• Can represent word problems through figures.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
					rectangle, parallelogram, triangle, rhombus and circle. • Applies the concept of area of rectangle, circle. • Can find the area of the rectangular paths circular paths.	
Data Handling	• Collection and organize of data • Mean, medium & mode, double • Bar graphs, simple pie charts	• Can organize the raw data into classifies data • Can solve the problems for finding the mean, medium, mode of ungrouped data and what they represent.	• Can understand the mean, mode and medium of un groups data and what they represent	• Can explain the mean, mode and median of ungrouped data	• Can apply the concepts of mean, median, and mode in solving the daily life problems. • Can understand the usage of double graphs and pie graphs in daily life situation(year wise population, budget, production of crop etc)	• Can represent the mean, medium and mode for ungrouped data. • Can represent given data in double bargraphs and simple pie graphs.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : VIII MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
1) Number System.	• Playing with numbers • Rational numbers • Square Numbers • Cube numbers • Square roots, cube roots, cubes.	• Pupil can expand 3digti numbers in the expanded form • Pupil can solve the number puzzle • Pupil can find the factors multiples of a given number • Pupil can solve the problems with divisibility rules (2,3,4,5,6,7,8,9,10,11) for a 2 and 3 digit number which is expressed in general form. • Can solve the problems (word / numeral) on rational numbers • Can solve the problems on rational numbers in the form of non terminating recurring decimal (vice versa) • Can find perfect square in between two given numbers.	• Can understand and write 2 and 3 digit numbers in generalized form. • Verify the results and finding errors in solving problems through divisibility rules and rectify it. • Can understand and verify properties at rational numbers • Can understand there are infinite rational numbers in between rational numbers. • Can guess and give reasons whether the given numbers are perfect squares or not. • Can understand the properties square numbers and verify • Can identify the patterns in square numbers and extend the given patterns	• Can explain the divisibility rules (2,3,4,5,6,7,8,9,10, 11) • Can convert express rational numbers as decimal number (vice versa) • Can give explain for square numbers, cube numbers, and Pythagorean triplets. • Can write express the square and cubes in exponential form. • Can use appropriate symbols ( ) for square root and cube root.	• Can solve the puzzles involving various concepts and operations. • Can solve the problems on rational number involving various concepts and operations. • Can find square roots through subtraction of successive odd numbers and through prime factorization method and by division method. • Can find the cube roots	• Can represents the rational numbers on number line.

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
		• Can find the square roots of a given number • Can find the cube roots of a given number	• Can verify that whether the given numbers are pythogorian triplets or not • Can estimate square roots of non perfect square numbers • Can verify that whether the given number is perfect cube or not? • Can understand the patterns of cubes and extend the given pattern and make generatization. • Can estimate the cube root of a given number		through adding consecutive odd numbers.	
Algebra	• Exponents & Power • Algebraic expressions • Linear equations in one variable • Factorization	• Can solve the problems by using laws of exponents • Can solve the problems on addition, subtraction of algebraic expression • Can solve the problems on multiplication of algebraic expressions (monomial, monomial: monomial, bionomial, bionomial, bionomial –	• Can understand laws of exponents and verify them. • Can simplify exponential form and give reasons. • Can compare smaller and very large numbers using the concept of same exponents.	• Can explain the terms base and exponent • Can give / say base and exponent of a given expression • Can express very small numbers in standard form using negative exponents	• Can solve the problems of exponents involving various concepts / operation (ex: through algebraic expressions etc) • Can verify the algebraic	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		into) • Can solve the problems by using algebraic identities • Can solve the problems on simple equations (variable having the one side and also two sides) • Can reduce equation to simpler form to linear form and solve problems based on it.	• Can differentiate like terms and unlike term from the given algebraic expression • Can verify the results and give reasons of problems on algebraic expression. • Can verify the algebraic identities	• Can convert the numbers standard form to exponential form vice versa. • Express the algebraic identities and explain them. • Can give examples for linear equations in one variable.	identities with geometrical concepts. • Can solve the problems on linear equations using algebraic concepts, arithmetic concepts and geometrical concepts.	
Arithmetic	• Comparing quantities using proportion • Direct and inverse proportion	• Pupils can find the compound ration of the given simple ratios • Can solve the word problems on compound ratio. • Can solve the problems on percentage, profit, loss, overhead expenses, discount and tax. • Can solve problems on simple interest and compound interest • Can solve the word problems on direct proportion and also inverse proportion.	• Can estimate percentages, profit, discount etc in a given situation / problem and give reasons. • Can differentiate between simple and compound interest. • Can give the reasons for the solutions problems related to direct, inverse proportions time and work, time and distance.	• Can give examples for compound ratio from daily life. • Can explain the formula for percentage, loss, profit, discount, simple interest and compound interest. • Can create new problems on their own for the given concepts (i.e ratios and proportions)	• Can solve the problems involving applications on percentage, profit, loss, discount and tax, using more concepts / operations. • Can solve the problems on direct, inverse proportion (mixed problems) compound proportion.	

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
		• Can solve the problems on time & work, and time & distance.				
Geometry	• Construction of quadrilaterals • Represents 3D and 2D • Exploring geometric figures.	• Can identify the edges, faces, and vertices of given 3D shape. • Can solve the problems on 3D shapes (related to finding to area, finding measurements etc.) • Can find the number of unit cubes in 3D figure on dot paper.	• Can give reason whether he / she construct the quadrilateral for given measurements. • Can give reasons for selecting suitable 2D-net for given 3D-shapes.	• Can explain the properties of quadrilaterals • Can explain the steps in construction of quadrilaterals • Explain about 3d shapes according faces, edges, verifies. • Can give examples for 3D- shapes objects (cube, cuboid, poly hydren etc) • Can explain eular's formula for poly hydren.	• Can use concepts of triangles in construction of quadrilaterals • Can use various geometrical concepts (area and perimeter of a square, real circle triangle) in solving problems on 3D shapes.	• Can construct quadrilaterals with given measurements • Can draw 3D objects on 2D isometric dot paper • Can draw the net shapes (2D shape) for the given 3D shape)
Mensuration	• Area of plane figures • Surface areas and volumes (cube, cuboid)	• Can solve the problems related area of quadrilaterals (Trapezium, rhombus, parallelogram etc.) • Can solve the problems on area of circle and circular paths semi circle, sectors and length of arc.	• Can give reasons whether the given quadrilateral can be divided into two consume triangles or not. • Can generalize the area of circle through various activities / methods	• Can explain the formula for area of quadrilaterals (Trepizium, rhombus etc) circle sector, triangle, square and rectangle.	• Can solve the problems on area of plane figures by using various concepts / operations (geometrical and algebraical concepts)	• Can drawn and represent quadrilaterals (square, rectangle, rhombus, trapeguim etc) on the graph paper.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can solve the problems on surface area of cube and cuboid and also volume	• Can identify the relationship between total surface area and lateral surface area. • Can compare and contrast TSA – LSA, volume of cube and cuboid also.	• Can give examples for cube and cuboid • Can explain the formulae of surface are and volume of cube and cuboid.	• Can use algebraic concepts and geometrical concepts in solving the problems on surface area and volume of cube and cuboid (with more operation also)	
Geometry	• Exploring geometric figures	• Can solve the word problems related to congruency and similarity	• Can identify the congruency of given reasons. • Can check the similarity of given figures. • Can differentiate between congruency and similarity,	• Can explain congruency of two figures, similarity of given figures. • Can explain the process in constructing dilation for given figure. • Can explain about lines of symmetry give examples of figures, may have morethan one line of symmetry • Can explain about line symmetry rotations symmetry and point symmetry.	• Can solve problems related height, distance etc by using various geometrical concepts	• Can represent the dilation of given figure on a graph paper. • Can construct the dilation figure of given figure. • Can draw lines of symmetry for a given figure.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
Data Handling	• Frequency distribution tables and graphs	• Can solve the problems on mean, median and mode of ungrouped data. • Can find the mean by deviation method. • Can interpret and construct of the grouped frequency distribution.	• Can estimate the arithmetic mean and median for ungrouped data and verify them. • Can draw the inferences of the data	• Can explain the terms and formula for mean, median and mode and their advantages and disadvantages. • Can explain scope and necessity of grouped data.	• Can use algebraic and arithmetic concepts in solving problems related to mean. Mode and median of ungrouped and grouped data. • Can relate mean, media, mode with each other while problem solving.	• Can prepare frequency distribution tables for given data. • Can prepare cumulative frequency distribution tables for given data. • Can represent the data in frequency graphs (Histogram, frequency polygon, frequency curve, cumulative frequency curves).

Mathematical Communication is precise and employs unambiguous use of language and rigout in formulation, which are important characteristics, conscious and stylized. Mathematicians discuss what is appropriate notation since good notation is held in high esteem and believed to aid thought. As children grow older, they should be taught to appreciate the significance of such. conventions and their use. For instance, this means that setting up of equations should get as much coverage as solving them.

- NCF 2005

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : IX MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
NUMBER SYSYEM	• Real Numbers	• Pupils can find rational numbers between given two rational numbers. • Can solve the problems on real numbers. (Rationalizing the denominators, by using its conjugate or rationalizing factor.)	• Can compare the given numbers (rational / irrational) and give reasons. • Can differentiate rational and irrational numbers.	• Can express rational numbers in decimal forms. • Can give examples for rational / irrational numbers / surds.		• Pupils can represent terminating / non-terminating recuing decimals on the number line through successive magnification. • Can represent rational & irrational numbers on number line.
ALGEBRA	• Polynomials • Linear equations in two variables	• Can solve the problems on polynomials (Finding the value of variable, finding zeros of polynomials, division and factorization of polynomial) • Can solve the problems on polynomials by using reminder theorem & factor theorem. • Can solve the linear equations in two variables.	• Can differentiate various polynomials (monomials, binomials) and give examples. • Can verify the division & factorization of a polynomial by using reminder theorem and factor theorem. • Can verify algebraic identifies. • Can verify the solutions of given linear equations.	• Can express and explain monomial, binomial, trinomial etc., according to the no. of terms in it, and give examples for the above. • Can explain about reminder theorem and factor theorem. • Can identify and explain the linear equations in two variables.	• Can solve the problems of day to day life by using linear equations (Bu using arithmetic, algebraic concepts)	• Can represent linear equations in two variables on graphs (plane) and read the graph. • Can draw the equation of lines parallel to X-axis and Y-axis.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
				• Can write daily life situations in the form of linear equations (Vice-versa).		
CO-ORDINATE GEOMETRY	• Cartesian system • Plotting points in a co-ordinate plane		• Can justify the position of points in a co-ordinate plane.	• Can say abscissa and ordinate of a given point and also as X-co-ordinate, Y-co-ordinate. • Can express the point using brockets (i.e. : (x, y))	• Can find the areas of given geometrical shapes joining the points in a plane (using graph).	• Can locate a point in the co-ordinate plance.
GEOMETRY	• Elements of geometry • Lines and angles • Triangles • Quadrilaterals • Area • Circles • Geometrical constructions	• Can solve the problems on linear pair of angles	• Can prove theorms (Ex : Two distinct lines cannot have more than one point in common") • Can show and verify the axioms & postulates for given situations. • Can differentiate between intersecting lines and concurrent lines. • Can prove (The sum of angles of a triangle is 180^0") theorems with lines and angles, lines of transversal etc.	• Can give axioms from their day to day life. • Can appreciate Euclidian geometry, axioms and postulates. • Can identify and explain intersecting lines and non-intersecting lines. • Can identify and say various types of angles. • Can explain about linear pair of angles.	• Can solve problems on angles involving arithmetic and algebraic concepts. • Can solve problems on lines of transversal	• Can draw geometrical figures with given measurements.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can solve the problems based on congruency of triangles. • Can solve problems parallelogram • Can solve problems on area of triangle and quadrilaterals.	• Can give reasons in constructions of geometrical figures. • Can give reasons for congruency in triangles and inequalities in a triangle. • Can prove theorems on based on congruency of triangle. • Can verify rules of congruency of triangle (i.e, SAS, AAS, SSS, ASA, RHS etc.) • Can verify and give reasons in finding angles in parallelogram. • Can show and verify the given statements based on parallelogram and the mid point theorem of triangle. • Can prove theorems ("Parallelograms on the same base and	• Can explain the steps in construction geometrical figures. • Can explain congruency of triangles and rules of congruency in triangle. • Can explain properties of parallelogram. • Can differentiate between congruency and similarity.	using different concepts. • Connect the rules of congruency to the other plane figures.	• Can construct triangle with given measurements & circle segment also. • Represent the plane figure formed when the mid points of the sides of the given figure are joined.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
		• Can solve problems on angle subtended by chord at a point in a circle. • Can solve the problem on angles made by major arc and minor arc of a circle. • Can solve problems on angles of a cycle quadrilaterals.	between the same parallels are equal in area" etc) and verify it. • Can prove theorems based on angle sustended by a chord at a point in a circle. • Can give reasons for determining angles made by a chord, major are minor arc and cycle quadrilaterals.	• Can give statements in mathematical language on conclusions he made. • Can explain steps in construction of a) Perpendicular bisector of a	• Can connect the concepts of angle made by a chord on the circle and angle made by a arc. • Can connect the generalization he made on angles made in different situations while problems solving.	• Can construct circum circle with given measurements. • Can construct a) Perpendicular bisector of a given line

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
				given line segment. b) Bisector of a given angle. c) and a triangle.		segment. b) Bisector of a given angle.
MENSURATION	• Surface areas and volumes	• Can solve problems on surface areas & volume of cube, cuboid, prism, cylinder, cone, sphere, hemisphere and right circular cylinders / cones.	• Can compare and contrasts surface areas and volumes of cube, cuboid, prism, cylinder, cone, sphere, hemisphere etc.	• Can explain formulae for surface areas and volumes of cube, cuboid, prism, cylinder, cone, sphere, hemisphere etc. • Can explain the relationship of surface areas of given figures and also for volumes of given figures.	• Can solve the problems on surface areas, volumes of cube, cuboid, prism, cylinder, cone, sphere, hemisphere etc. by sing algebraic, arithmetic concepts and more operations. • Can relate / connect area and volume of cube cuboid, prism, cylinder, cone sphere, hemisphere to each other infinding surface areas and volumes of 3D objects.	• Can represent 3D figures (cube, cuboid, cylinder, etc) in 2D figures (net shapes)

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
STATISTICS AND PROBABILITY	• Statistics	• Can calculate mean, median, mode of ungrouped date. • Can find the mean by deviation method.	• Can give reasons for their judgments on mean, median, mode. • Can estimate mean, median for ungrouped date and verify them.	• Can explain the terms and formulae for mean, median, and mode and their advantages and disadvantages. • Can explain scope and necessity of grouped date.	• Can relate mean, median, mode with each other while problem solving. • Can we algebraic and arithmetic concepts in solving problems related to mean, median, mode.	• Can prepare frequency distribution / cumulative frequency distribution tables for given data. • Can represent the data in frequency graphs (Histograms)
	• Probability	• Can solve the problems on single event of a sample space of a random experiment.	• Can estimate probability of an event of a random experiment and give reasons in finding probability.	• Can explain the terms random experiment event, probability etc., • Can explain the statements related to probability in mathematical language.	• Can connect the previous arithmetic concepts to understand and solving problems.	• Can represent the outcomes of the random experiment in the form of table.
PROOF IN MATHE MATICS			• Can generalize the observations into mathematical statements on the basis of inductive and deductive reasonings.	• Can express their generalizations in mathematical statements.	• Can connect their generalize-tions to form new statements.	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
			• Can give reasons for different statements made by them. • Can prove the mathematical statements and verify them.			

An open-ended problem may yield multiple answers. Such a problem, requiring divergent thinking, may be solved by many different methods. There will be a great need for investigative and reflective thinking and decision making, to justify the process and the product.

Open-ended questions are not to be confused with 'opening' questions. Opening questions are simply starting points to probe into the background knowledge in the topic to be introduced, the past experiences and the recall of the learner. They are mostly closed-ended. However experience tells us that commencing a class with open-ended questions can spark mathematical communication.

A teacher should use a judicious combination of closed-ended and open-ended questions. Closed-ended questions alone may not provide a real assessment in instruction. It is necessary for the teacher to wait for the responses of the students when an open-ended question is asked, and not to hurry the student. Without this allowance of time, the teacher may miss opportunities to sport learning difficulties as well as patterns of valid but divergent thinking in the learners.

ACADEMIC STANDARDS AND LEARNING INDICATORS

CLASS : X MATHS

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
NUMBER SYSYEM	• Real Numbers	• Pupils can solve the problems on finding LCM, HCF by using prime factorization method. • Can solve the problems on rational numbers, irrational numbers and logarithms.	• Pupils can understand and verify the fundamental theorem of arithmetic and give conclusions for the above theorem. • Can prove the results of irrationality of$\sqrt{2}$, $\sqrt{3}$ etc., and decimal expansion of rational numbers interms of terminating / non terminating recoring decimal (vice. versa) • Can understand and verify the properties of real numbers. • Can prove and verify laws of logarithms through generalizations made by them by inductive reasoning & deductive reasoning.	• Can give examples and explain the fundamental theorem of arithmetic. • Can state whether the given rational numbers will have terminating / non terminating, repeating decimal form by without performing division. • Can convert the given number in exponential form to logarithm form (vice versa) • Can explain the scope and necessity of logarithm and can explain laws of logarithms and terms in logarithms.	• Can connect some concepts of real numbers in solving problems. • Can connect laws of expands to laws logarithms and also each law of logarithms to derive other laws of logarithm. • Can connect of logarithm in daily life situations.	• Can represent a red number on a number line.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Sets	• Can solve the problems related to sets and their basic operations. (Union, interest, difference)	• Can differentiate empty set, finite set, infinite sets and universal sets and give reasons. • Can justify whether the given statements belong to set or not. • Can identify equal sets and give reasons. • Can justify that the inter section of two disjoint sets is a null set.	• Can give examples for sets. • Can express the given set in roster form and set builder form. • Can convert roster form of a set to set builder form (vice-versa). • Can use the signs / symbols regarding sets at the appropriate time / situation. • Can explain about subsets.	• Can connect the concept of set in daily life situations. • Can interlink number, arithmetic algebraic concepts in sets.	
ALGEBRA	• Progression	• Can find the required term of given Arithmetic progression and also find common difference of A.P. • Can calculate the n^{th} term and sum of first n terms of an A.P. • Can calculate the n^{th} term in G.P.	• Can give generalizations from Arithmetic progression, Geometric progression for common difference in AP and common ratio in GP and n^{th} term in AP and GP. • Can generate the formula for sum of n terms in AP.	• Can express and explain the general form of AP, GP. • Can explain the formula and terms in AP and GP. • Can give examples for AP and GP.	• Can connect arithmetic and algebraic concepts while solving the problems on AP and GP.	

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Polynomials	• Can find the zeros of polynomial (quadratic polynomial and cubic polynomial) • Can solve the simple problems on division algorithm for polynomials with Integral co-efficients.	• Can verify / check and give for reasons zeros of a given polynomial. • Can made some generalizations by observing zeroes of polynomials and justify them.	• Can express "degree" of a polynomial, zeroes of a polynomial terms of the polynomial (linear, quadratic & cubic polynomial)	• Can connect / interlink the zeroes and coefficients of a polynomial to each other whole solving problems.	• Can represent a linear polynomial on the graph and also quadratic polynomial, cubic polynomial.
	• Pair of linear equations	• Can find solutions on pair of linear equations in two variables. (Model methods algebraic method, elimination method) and substation method. • Can solves the simple problems on linear equations.	• Can verify and given reasons for solutions of pain of linear equations. • Can check whether the given pair of linear equation is consistent or not and dependent and can give reasons.	• Can express their daily life situational issues in the form pair linear in two variables. (Vice-versa).	• Can relate between co-efficients and nature of system of equations. • Can solve the problems of different situations (by using algebraic and arithmetic and geometrical concepts) from day to day life related to pair of linear equations.	• Can represent the pair of linear equations in two variables through graph. And find / identify the solutions for them in graphical method.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Quadratic equation	• Can solve the simple problems on quadratic equations by using factorization, and the method of completing the square.	• Can give reasons for the solutions (nature roots) for a quadratic equation. • Can verify that whether the solutions of a given quadratic equation are correct or not (or) Can verify that whether given solutions are roots of the quadratic equation and give reasons for that. • Can estimate the roots of quadratic equations and give reasons through verifying them.	• Can express day to day life situations in the form of equations (Vice-versa). • Can express the nature of the roots of quadratic equations.	• Can solve the problems on quadratic equations involving algebraic, geometric concepts.	• Can draw the graph for quadratic equations.
GEOMETRY	• Similar triangles	• Can solve the problem based on theorems, (Thales theorem etc.) (properties of similar triangles) • Can find the area of similar triangles. • Can solve the problems on Pythagoras theorem.	• Can prove theorems based on similarity of triangles. • Can made the conclusions that all the congruent figures are similar but the converse is not true. • Can prove & give converse for the theorems by examining through some activities. • Can prove theorems related to similarity properties of triangle.	• Can give statements by their generalizations in mathematical form. • Can explain properties of triangles. • Can give converse statement, negation of statement for given statements.	• Can connect Algebraic and various geometrical concepts in solving the problems in different daily life situations.	• Can draw the line segment and division of that wish-given measurements. • Can draw triangles with given measurements.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Co-ordinate geometry	• Can calculate distance between two points a co-ordinate plane. • Can find area, perimeter of given geometrics figures joining points in plane. • Can solve the problems on section formula. (dividing line in two segments with a point in given ratio) • Can find the tri-sectional points of a line segment. • Can solve the problems on finding mid point, centroid and slope of a line.	• Can give the reasons for the location of a plotted points in a coordinate plane and for the distance between those points. • Can generalize the mid point of a line segment through some activities related section formula. • Can generalize the slope of a line and can give reasons for its. (Based on angle (inclined) of a line a plane.)	• Can explain the formulae for distance between two points in a plane and the area of a figure formed by joining pints in a plane etc.. • Can express their conclusions had made from co-ordinate geometry in mathematical statements (Vice-versa)	• Can connect various geometrical concepts in solving problems on coordinate geometry. (Ex: perimeter, area of circle, triangle, Quadrilaterals etc.) • Can use Heron's formula to find the area of a given triangle joining three points in a plane. • Can connect the concept of linear equations to co-ordinate geometry. (i.e. straight line)	• Can plot the given pints on a coordinate plane. • Can draw the figures by joining given points in a plane.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
	• Tangents and secants to a circle	• Can solve the problems in finding length of the tangent of a circle. • Can solve the problems in finding the area of the segment.	• They can made conclusions on tangents / secants of a circle and can give reasons. • Can distinguish and differentiate tangents and secants of a circle. • Can prove theorems on tangents and secants of a circle and their converse also. • Can made generalization and differentiate area of the minor / major segment of a circle.	• Can explain about tangents and secants of a circle. • Can explain theorems / statements in his own and in mathematical statements.	• Can connect the idea of the chord in understanding tangent and secants of a circle. • Can connect various geometrical concepts in solving tangents and secants of a circle, and area of the segment in the circle.	• Can draw / construct tangents at different points of a circle. • Can draw / construct secants of the circle. • Can draw a pair of tangents of circle.
TRIGONO METRY	• Trigonometric ratios	• Can solve the problems on trigonometric ratios for the angles from 0^0 to 90^0. • Can solve simple problems on trigonometric identities.	• Can give reasons in finding values of trigonometric ratios from 0^0 to 90^0 and lengths of a line etc. in given triangle. • Can generalize trigonometric ratios and can verify them.	• Can explain the terms “Hypotenuse, opposite side, adjacent side of given triangle. • Can explain the terms sin, cos, tan etc and using by them they can give their conclusions in mathematical statements.	• Can use Algebraic concepts while solving problems on trigonometric ratios.	• Can prepared a table of for trigonometric ratios for the angles from 0^0 to 90^0.

Area	Key concepts	AS_1 (Problem Solving)	AS_2 (Reason & Proof)	AS_3 (Communication)	AS_4 (Connection)	AS_5 (Representation)
				• Can express the scope and need of trigonometry (in solving day to day life problems) in mathematics.		
	• Applications of trigonometry	• Can solve simple problems on application of trigonometry. (hights, distance etc).	• Can differentiate the angle of elevation, angle of depression in a triangle (while solving problems)	• Can explain and give their conclusions of angle of elevation, angle of depression in mathematical statements.	• Can solve the daily life problems by using trigonometry. (by connecting Algebraic and geometrical concepts)	• Can draw figures related to problems on trigonometry.
PROBABI LITY	• Probability of single event	• Can solve the problems on simple space of a random experiment in different methods.	• Can give conclusions and generalizations by experimentally and theoretical on probability on single event and sure event, impossible event. • Can assume and gives reasons of equally likely outcomes.	• Can explain terms in probability. • Can express the use of probability.	• Can connect the pervious arithmetic concepts to understand and solving problems on probability.	• Can give the outcomes or represent out comes of the random experiment in the form of table.
MATHE MATICAL MODELLING	• Mathematical models		• Can give reasons for steps in solving problems and in modeling	• Can give some mathematical models for previous class that they already learnt.	• Can inter link various concepts for modeling.	

Area	Key concepts	AS$_1$ (Problem Solving)	AS$_2$ (Reason & Proof)	AS$_3$ (Communication)	AS$_4$ (Connection)	AS$_5$ (Representation)
				• Can express the advantages / limitations of mathematical modeling.		
MENSURA TION	• Surface areas and volumes	• Can solve the problems on finding surface area, volumes of combinations of any two of the given solid shapes.	• Can give generalizations and can concludes through area, volume of simple solid shapes to each other and to combination of two solid shapes and give reasons for them.	• Can explain terms and formulae in mensurations areas, volumes of various solid shapes.	• Can use various geometrical, algebraic, arithmetic concepts in solving problems on mensuration.	• Can draw simple solid shapes and combination of solid shapes with given shapes.
DATA HANDLING	• Statistics mean, median, mode	• Can solve simple problems on finding mean, median, mode for given ungrouped / grouped data with different methods.	• Can estimate mean, median, mode for given ungrouped data and can give reasons. • Can distinguish he ogives boundaries	• Can explain the terms mean, median, mode grouped data, ungrouped data. • Can explain usage of different values of central tendencies through ogives. • Can explain the terms in the formulae.		• Can represent the data in the form of distributive / cumulative frequency tables. • Can represent the data in graphical way.

7 - MATHEMATICS – CURRICULUM IMPLEMENTATION – LEVEL: CLASSES I AND II

(Teaching Learning Process, TLM, Text Books, Teacher Preparation and Evaluation)

Introduction :

Every learner (student) is important in the classroom. Unlike the present classroom in which only few students performance is taken as success or failure, performance of every student should be Ensured. Classroom environment, teaching learning process, teaching learning materials, textbooks, teacher's readiness & preparation and evaluation play important role to make a classroom effective.

We are under misconception that a learner's mind is a blank slate and whatever we want can write or erase on it. Before joining in the school, a student has lot of notions of mathematics. These notions are very important contrivances to achieve academic standards at the level of 1 and 2 classes let's discuss about important factors that effect a mathematics classroom.

- **Classroom Environment – Teaching Learning Processes**

The classroom Environment and the organization of teaching learning processes should be on the lines discussed hereunder.

- The classroom Environment should be so pleasant that the children participate in various activities freely.
- The children should not be seated under trees or verandahs. They should be comfortably seated in well ventilated big classrooms.
- Care should be taken that there is no fear of punishment, prohibition, reproach, or stressful Environment in the classroom.
- The classroom should have facilities to conduct individual, group and whole class activities.
- The seating arrangement should not be rigid but flexible. The children can be seated in a circle or a semi-circle or in any other way to facilitate the organization of activities.
- The classroom Environment should be conducive to free expression, questioning and discussion among children.
- The teaching learning activities should definitely be textbook-based and the teacher should use other material (Snehabala Cards) to supplement textbook-based teaching.
- Ample time should be given for the children to question and to participate in various teaching learning processes regularly.
- The number concept activities should be organized in a phased manner - first by encouraging the children to count various objects (in ones and tens) and then by making them read and write numbers.
- In each unit, the teacher should facilitate good understanding and see that all the children in the class do the whole class activities first, and then only proceed to do the problems in exercises individually.
- While organizing whole class activities, the teacher should first see that the children have a good understanding of the activities in Snehabala Cards too. Then, she should involve the children in the group activities suggested in the cards.

- Once all the exercises in a unit are completed, the teacher should evaluate the performance of the children using a question paper prepared by her (using the exercises in the unit) or by using the evaluation card in Snehabala Cards.
- Strings of beads, bundles of sticks, coins and currency notes, etc., should be used while conducting activities that develop the number concept and the four fundamental operations.
- Every day, the first 45 minutes (out of 85 minutes allocated for maths) should be spent for conducting activities taking the whole class into consideration. The next 40 minutes should be spent to organize teaching learning activities especially for low achievers or low performers.
- After understanding the concept, a model problem solving should be discussed on black board.
- During the 2nd session of 40 minutes, the exercises of "Do this", "Try this" and "Think & discuss" should be practiced by students.
- "Do this" should be solved individually by students.
- "Try this" should be solved individually or in groups.
- "Think & discuss" should be answered after discussion in groups.
- The problems in exercises should be given as homework. Everyday, the solved home work problems should be corrected. And misconceptions of students should be corrected.
- As a part of the exercises related to the number concept and the four fundamental operations, the teacher should definitely organize the games and puzzles suggested in Snehabala Cards.
- The use of grid paper, counting and colouring squares(checks), writing corresponding numbers, etc., should find place in classes 1and 2 teaching learning processes.
- The teacher should make sure that all the children in the class understand the language used by her in teaching learning processes. If necessary, she should first use the language of the children, and then only switch over to mathematical language or mathematical terminology.
- The teaching learning activities should be organized at least for 30 minutes every day, keeping in view the individual differences and the consequent multi level learning.
- While organizing teaching learning processes, the teacher should facilitate understanding by writing on the board, by questioning and by encouraging interaction and also should give opportunity for the child to think and question.
- While organizing teaching learning activities related to the number concept and the four fundamental operations, the teacher should spend 10 minutes to give practice in counting by making the children count nuts piece by piece as well as in heaps. She should also encourage them to read and write numbers.

- **Teaching Learning Materials:**

While teaching mathematics to classes 1 and 2, the teacher should keep the following material available. She should definitely use the material in organizing various teaching learning processes.

- Snehabaala Cards
- Strings of beads
- Nuts / pebbles
- Sticks / bundles
- Dummy currency, coins
- Dice, snakes and ladders, counters/pieces
- Puzzles, cards related to games
- Number cards (1 – 100) (Flash Cards)
- dot Cards(1 – 50) (Two 50 dot cards)
- Grid paper
- Various number tables
- Colour pencils, colour paper
- Cards with pictures
- Newspapers / Magazines
- Small wooden are plastic cubes in 1's, 10's and 100's.
- Calendar
- Measuring jars, weights, ribbon tape, etc.
- Models of basic 2-D geometrical figures.

- **Textbooks:**

The textbooks for classes 1 and 2 should have the following qualities.

- The textbook serve as a textbook cum workbook.
- The design of the textbooks based on the expected competencies of the children and facilitates the achievement of grade-appropriate objectives of teaching mathematics.
- The textbooks are with quality paper with colourful pictures, eye-friendly font, quality print and attractive to the young mathematicians.
- The textbook make the children to think and it contain a lot of songs, games, colouring, and such other activities which encourage thinking in an enjoyable way.

- The pictures in the textbook help the children understand the concept, and also promote their understanding of the instructions and activities given in it.
- The textbook uses real life situations or a story or a song to facilitate the understanding of a concept which helps the children to solve problems and acquire mathematical skills.
- The pictures and activities in the textbooks should make the children think.
- The textbook should use child-friendly language and contain a lot of pictures. It should have a number of activities that require the children to count, match, tick (the correct answer), and solve problems applying their understanding of mathematical concepts.
- The exercises should be such that they enable the children to have a good understanding of the number concept and the addition and subtraction operations.
- The language used in the textbook should be intelligible to the children belonging to various areas in the state. The instructions should be so clear that they can be easily understood and followed when said or read.
- The textbook should encourage mathematical logical reasoning.
- The activities and exercises in the textbook should promote the culture of 'learning outside the classroom'.
- The textbook should strive for the optimum utilization of the natural interest and enthusiasm of the children in developing and promoting mathematical concepts.
- The units should be arranged in a spiral so that the same concept finds place in different units after a certain gap.

- **Teacher Readiness – performance indicators**
    - The teachers of classes 1 and 2 should take individual responsibility their class and prepare timetable accordingly.
    - Out of the 85 minutes allocated for maths 45 minutes should be spent on teaching and conducting activities. The next 40 minutes should be spent on organizing teaching learning processes specially for low achievers or low performers. In the first 45 minutes, 10 minutes can be spent on the understanding (recalling) of the previous mathematical concepts, 15 minutes can be spent on the introduction of new mathematical concepts, 15 minutes can be spent on practice and exercises, and the last 5 minutes can be spent on checking the comprehension/understanding of the concepts. The teacher should prepare this plan beforehand.
    - The teacher should prepare year and lesson / unit plans according to the monthly allocation given in the contents page of the textbook to facilitate the organization of the teaching learning processes.
    - The teacher should get ready with necessary teaching learning material like, a string of beads, sticks and bundles, number cards, nuts, geometrical figures, material required for measurement of solids and liquids, etc., before starting a unit or a lesson.
    - The teacher should check the activities done by students, give suitable instructions, and sign the worksheets/workbooks.
    - The teacher should take responsibility for the whole class for the achievement of the expected competencies in each unit.

- For this, the teacher should evaluate the performance of the class by administering the competency based test prepared by her, analyse the test results, and take up remedial teaching/measures.
- The teacher should see that all children participate in the teaching learning processes. Counting objects, identifying numbers, reading and writing numbers, etc., should definitely be done everyday.
- The teacher should attend in-service teacher training programmes and use that knowledge/skills in classroom teaching.
- The teacher should rearch for extra reading materials such as books, magazines, papers etc or internet columns or materials before teaching.

- **Evaluation**

Evaluation or assessment is an important factor which effects learning of a learner. Till recent past years, the evaluation process has been implemented to only test that the knowledge of the student. This evaluation has moved around marks and ranks only. This created lot of stress and strain on the minds of children.

But, according to views of experts, assessment is for learning, assessment while learning and assessment after learning.

At the level of 1 and 2 classes the assessment should be as it is discussed here.

- Evaluation should not be discontinuous and confined to test memory of students. It should be done continuous.
- The assessment should be comprehensive. It should assess the all expected competencies in mathematics.
- Assessment should be done in 2 types i.e. formative and summative.
- In formative assessment counting skills, comparison skills, language skills, exploration, reasoning skills etc. should be assessed while learning.
- Assessment of doing addition, subtraction and multiplication with reasoning should be done.
- In summative assessment 20% of it should be conducted orally and the remaining should be conducted in written form.
- In summative assessment 50% of weightage is given to problem solving.
- In formative assessment, classroom responses, written works, project works, slip tests are tools to assess the learning.
- Slip tests should not be confined to test the memory of students but it should assess the competencies in mathematics.
- Slip tests are not periodical tests. They can be conducted random test.

8. MATHEMATICS – CURRICULUM IMPLEMENTATION – LEVEL: CLASSES III, IV AND V

(Teaching Learning Process, TLM, Text Books, Teacher Preparation and Evaluation)

Introduction :

Every learner (student) is important in the classroom. Unlike the present classroom in which only few students performance is taken as success or failure, performance of every student should be Ensured. Classroom environment, teaching learning process, teaching learning materials, textbooks, teacher's readiness & preparation and evaluation play important role to make a classroom effective.

At the standard of I and II classes, students learn fundamental knowledge about numbers and operations. The students at the standard of III, IV and V Classes continue to learn about larger numbers and operation on them. They also continue to learn about basic geometrical 2-D figures, areas, perimeter etc. Find they even learn basic statistical skills.

To achieve mathematical competencies on above concepts, the necessary things are discussed here.

a. Classroom Environment – Teaching Learning Processes

The classroom environment and the organization of teaching learning processes in classes 3,4 and 5 should be on the lines discussed hereunder.

- There should be a pleasant and free environment in the classroom.
- The children should have textbooks as well as notebooks.
- The teacher should see that all children participate in the teaching learning process. She should see that they talk freely, express their ideas without any inhibition or fear and question whenever necessary. But she should not condemn their opinions/ideas/proposals, or compare one child with another, or demean them.
- Whole class activities, group activities, and individual activities should find place in the teaching learning processes.
- The activities provided should contain the nature of inductive reasoning.
- The children should be encouraged to do the textbook exercises on their own.
- The activities should be so designed and organized that they enable the children to understand the concepts in the textbook, to think logically, and to express mathematically(using mathematical terms appropriately).
- The teacher should make use of real life situations to help children comprehend mathematical concepts easily. She should introduce concepts contextually, encourage the children to discuss them. Similarly, learning should be done through games, mathematical diversions, puzzles, etc.
- The teacher should see that the children make notes of what they do in groups. She should also check them regularly.

- The teacher should see that the children use maths materials (maths kit) during the teaching learning processes.
- After understanding the concept, a model problem solving should be discussed on black board.
- During the 2nd session of 40 minutes, the exercises of “Do this”, “Try this” and “Think & discuss” should be practiced by students.
- “Do this” should be solved individually by students.
- “Try this” should be solved individually or in groups.
- “Think & discuss” should be answered after discussion in groups.
- The problems in exercises should be given as homework. Everyday, the solved home work problems should be corrected. And misconceptions of students should be corrected.

b. Teaching Learning Materials:

The following material should be available in the maths classroom

- Strings of beads, sticks – bundles, marbles, nuts, dummy currency – coins
- Calendar, tape, measuring cups, simple balance and weights
- Mathematical instruments box
- Cubic rods, geo-board, peg board, fractions disk
- Grid paper, colour pencils
- Books of mathematical puzzles, stories, and games
- Number charts, flash cards, dice, domino cards, etc.
- Wooden or plastic cubes in 1’s, 10’s, 100’s

c. Textbooks:

While developing textbooks for classes 3,4, and 5, the authors should keep in view the following elements.

- The textbooks should make the children think critically, and give space for practice and mathematical expression.
- The pictures in the textbook should facilitate comprehension of concepts and help children participate in various activities. They should be meaningful and attractive with eye-catching colours.
- The third class textbook should be 50% workbook. The textbooks for classes 4 and 5 should be 30% workbook.
- The chapters in textbooks should be related to numbers(different types of number, the four fundamental operations, decimal fractions), geometrical concepts, area – circumference, Data handling, etc.
- By the time the children finish classes 3,4, and 5, the respective textbooks should enable them to achieve mathematical skills, competencies, and oral mathematics of the corresponding class. They should also enable them to tackle real-life problems.
- The language of the textbook should be easy to understand and the activities should make the children think.

- The class 3 textbook should revise the foundational concepts of classes 1 and 2 before introducing new concepts. This procedure should be followed in classes 4 and 5 too.
- The exercises in the textbooks should not be stereotyped and mechanical. They should give scope for application of mathematical concepts, different solutions for the same problem, identification and remediation of mistakes, explanation of the chosen method and the method adopted, and mathematical expression.
- Mathematical puzzles and diversions, games, and projects should find place in the textbooks.
- Guidelines to use the textbook and the expected outcomes of each unit should be given in the textbook.
- The children's learning should not be restricted to the textbook but should go beyond it and enable them to refer and use other materials too.
- The size of the textbook should be convenient to handle and use. The cover page should be in attractive and colourful pictures.
- The units in the textbook should be so arranged that the same concept recur in various units in a spiral.
- The textbook should give space for the children to use various strategies in solving problems, and find more than one solution to problems.
- The textbook should give opportunities for the children to observe, identify patterns, generalize, continue the pattern by prediction, and discover or make new patterns.
- The textbook should give ample opportunities for the children to create / make new problems.
- The textbook should promote logical reasoning (specific for mathematics).
- The textbook should not give priority to definitions and explanations. It should promote discovery learning by which the children understand and learn concepts, and give definitions and explanations in their own words.

d. Teacher Readiness – performance indicators

- The teacher should read the textbooks intended for classes 3,4, and 5 thoroughly and develop comprehension of the subject and the activities.
- The teacher should get herself ready with month-wise plan (according to the annual plan given) and teaching learning materials.
- The teacher should first facilitate good understanding of the concepts by whole class activities. Only when she is sure of their comprehension, she should see that the children do the exercises in their notebooks individually.
- The teacher should see that all children participate in the teaching learning activities. She should see that they talk freely, express their ideas without any inhibition or fear and question whenever necessary.
- It is imperative that both the teacher and the children use teaching learning materials.
- In mathematics session, the teaching learning processes should be organized for 40 minutes every day keeping in view the individual differences and the consequent multi level learning.

- The teacher should make notes in her diary lesson plan about the activities given to each child, the expected outcomes, the no of children who achieved/ not achieved the targeted competencies and other important details to monitor the teaching learning process.
- Every day, meaningful exercises/work should be given to the children so that the time allocated for mathematics is put to good use.
- The teacher should check the activities/work done by students and their notebooks and give suitable suggestions /instructions.
- The drawings of the children, their solutions to problems, their solutions to puzzles, etc. should be displayed on the wall magazine.
- The teacher should attend in-service teacher training programmes and use that knowledge/skills in classroom teaching.

e. Evaluation

At the level of 3rd, 4th and 5th classes students continue to learn about larger numbers and their operations, basic geometrical shapes and their properties, and basic statistical skills. At this level problem solving, reasoning, connecting representation skills are developed. Even mathematical language of the student is improve. These skills in the said areas should be assessed continuously and enhance the students learn.

The process of assessment at this level should be like this.

- The assessment of the child should be comprehensive which assess problem solving, reasoning, connecting, communication, representation skills.
- The students should be assessed on number sense, spatial understanding and statistics at their respective levels.
- Assessment should be done in two types (i.e., formative, summative assessment).
- Formative assessment is done regularly while learning of students with tools i.e. classroom responses, written works, project works, slip tests.
- Slip tests are not periodical tests. They can be conducted randomly assess expected out comes.
- Teacher should prepare the questions paper her-self and conduct the Summative Assessment.
- The summative tests should be conducted in the written form. And they should be conducted in thries.
- In summative assessment, 50% of assessment is given for problem solving.

9. MATHEMATICS – CURRICULUM IMPLEMENTATION – LEVEL: CLASSES VI, VII AND VIII

(Teaching Learning Process, TLM, Text Books, Teacher Preparation and Evaluation)

Introduction :

Students of 6^{th}, 7^{th} and 8^{th} classes have the knowledge of numbers upto one lakh and spatial understanding of area and perimeter of different 2D shapes. They also know collection of data and organizing data in basic statistics.

At this level they have ability to group the different types of numbers into sets viz-natural, whole, integers etc… They can generalize and can express into a mathematical statements which needs variables. Therefore they have inner skills to learn algebra.

At this level they will learn types of numbers and their properties, arithmetic algebra, properties of 2D & 3D geometrical shapes, mensuration and statistics. They learn not only problem solving but also they acquire the skills of reasoning, connections and representations.

At this level teachers confined to transfer the information which is in the textbooks and they restrict them solves in explaining solving problems in the textbook only. But a successful mathematics classroom is expected to have the following qualities.

a. Classroom Environment – Teaching Learning Processes

The classroom environment and the organization of teaching learning processes in classes 6, 7 and 8 should be on the lines discussed hereunder.

- At the beginning of the upper primary level, the children should have the number concept of 4 digit numbers. They should be able to solve oral and written problems involving the four fundamental operations. Similarly, they should have a basic understanding of geometrical concepts and measurement. Making use of all these competencies/skills, they should be able to understand logical reasoning and explain concepts.
- The children should do the exercises given in the textbook on their own. This should be followed by discussion and explanation among the children.
- Discussion on various topics in mathematics (numbers, algebra, geometry, area, circumference, data handling), expression of opinions, and reaction and interaction on each others' opinions should be integral parts of teaching learning processes.
- To develop the competency of geometrical constructions and explanation of logical proofs, the teacher should facilitate comprehension using geometrical kit.
- The teacher should provide the children with opportunities to do the exercises in the textbook on their own. She should check them regularly.
- Solving puzzles, organization of quizzes, geometrical constructions, organizing projects, collection of mathematical puzzles and discussion on their solutions, etc., should definitely be in the teaching learning processes of a mathematics class.

- The teacher should see that the children make notes in their Note books about the mathematical topics they listen to, see, observe, discuss, or solve.
- The teacher should see that the children collect data on the mathematical concepts they learn in school from magazines, reference books, etc., and display them on the wall magazine.
- The teacher should use special material that facilitates learning while teaching special needs children. She should provide them with required special materials.
- The teacher should give children tasks involving collection of various types of information, tabulation and representation through graphs.
- Since algebra(which is more abstract) is introduced at this level for the first time, the teaching learning processes should be so designed and organized that they link this with the arithmetic which was already learnt.
- There is no space for discrimination among students in the teaching learning process.
- While teaching mathematical concepts, space should be given for discussion and explanation of their significance in real life context.
- Ample opportunities should be given to the children to understand the language of mathematics and comprehend the logical reasoning embedded in mathematical problems.
- Instead of solving problems using definitions and formulas, the children should be given opportunities to understand formulas and definitions on their own through discovery learning.
- The teaching learning processes should not encourage memorization of definitions / formulas, or solving problems taking the beaten path. They should enable the children to gain deeper understanding of the concepts, apply it to new situations, and comprehend the logical reasoning embedded in problems.
- The teacher should see that the children solve problems in their notebooks. She should check them and give suitable suggestions regularly.
- The teacher should see that the children ask questions without any fear. In the mean time, she should also see that they use their learning time productively.
- The teacher should make notes in her diary about the progress of the children, classroom management, feedback from children, etc. She should take special care of children with low achievement.
- The children should be given at least one project work/assignment in a month. The teacher should make the children bring out a report on their project, exhibit it and discuss it in the classroom.
- After understanding the concept, a model problem solving should be discussed on black board.
- During the 2nd session of 40 minutes, the exercises of “Do this”, “Try this” and “Think & discuss” should be practiced by students.
- “Do this” should be solved individually by students.
- “Try this” should be solved individually or in groups.

- “Think & discuss” should be answered after discussion in groups.
- The problems in exercises should be given as homework. Everyday, the solved home work problems should be corrected. And misconceptions of students should be corrected.
- The teacher should mark the answer scripts and analyze the mistakes and scores. She should discuss the mistakes/performance and take feedback from the children . Based on the observations and feedback she should take up remedial teaching.

b. Teaching Learning Materials:

While teaching mathematics to classes 6,7, and 8, the teacher should keep the following material available. She should definitely use this material in organizing various teaching learning activities.

- Reference books and magazines related to mathematics
- Grid paper, graph paper, ISO metric sheet
- Mathematical instruments box
- Modern electronic gadgets like TV, DVD, Tape Recorder, CDs, Computer, Internet, etc.
- Geo-board, two dimensional and three dimensional geometrical shapes and solids, Models.
- Maths kit.

c. Textbooks:

While designing textbooks for classes 6,7, and 8, the organization of units should be based on the points discussed hereunder.

- The textbooks should be attractive and the children should enjoy them. To achieve this, pictures, cartoons, stories, and other such material the children like should find place in the textbooks.
- The language in the textbooks should be so simple that the children can read and understand the books on their own. The teacher can only be a facilitator.
- Every new concept should be introduced linking it with the concept already learnt by the children.
- Various concepts in the textbook should be linked with one another.
- Since mathematics has scope for discoveries and inventions, the problems in the textbooks should not be like the casts from the same mould. Instead, there should be new challenges for the children as they go from one problem to another.
- The textbook should contain activities that give opportunities to children to reason logically.
- The textbook should give opportunities for the children to observe patterns, generalize, and continue the pattern by prediction.
- The textbook should give space for the children to use various strategies in solving problems and find more than one solution to problems.
- The exercises in the textbook should promote self learning and encourage the children to read other mathematics books.

- At this stage, way of proof should be given priority over solution. Hence, the concepts in the textbooks should be organized in a way that they give more weightage to proof.
- The textbook should encourage children to create new problems in groups as well as individually.
- Guidelines to use the textbook and the expected outcomes of each unit should be given in the textbook.
- The textbooks should suggest suitable projects that help children learn by direct experience.
- Puzzles and riddles should find their way into the textbooks to enable the children to get ready for the new challenges inside and outside the classroom.
- The textbooks should give historical background of concepts and details (biographical as well as academic) of mathematicians.

d. Teacher Readiness – performance indicators

- The teacher should prepare annual, unit, lesson, and period plans.
- The teacher should read the textbooks of classes 6,7, and 8 thoroughly and develop comprehension of the concepts and problems. She should first do all the exercises herself.
- The teacher should check the children's understanding of the previous mathematical concepts related to the ones in classes 6,7, and 8. She should extend her cooperation wherever necessary.
- The syllabus should be finished on time.
- The teacher should prepare the question papers herself and conduct the Summative Evaluation.
- The teacher should prepare or collect all necessary teaching learning materials beforehand.
- The teacher should see that all children achieve all competencies in each unit.
- The teacher should attend in-service teacher training programmes and use that knowledge/skills in classroom teaching.

e. Evaluation

- Apart from evaluation in the form of marks at discontinuous periods, the assessment should be continuous and comprehensive.
- In formative assessment the student should be assessed, continuously while learning with tools & classroom responses, written works, project works, slip test.
- This formative assessment should encourage the learning of student but not to discourage the students.
- The both assessments should assess the students for the skills of problem solving, reasoning, communications, connection and representation.
- The summative test should be conducted thries in a year in the written form.
- 40% of the assessment should be conducted for problem solving skills.
- The teacher should prepare the question paper herself and conduct the Summative Evaluation.

10. MATHEMATICS – CURRICULUM IMPLEMENTATION – LEVEL: CLASSES IX AND X

(Teaching Learning Process, TLM, Text Books, Teacher Preparation and Evaluation)

Introduction :

At the level of 9^{th} & 10^{th} classes students are at the end stage of secondary education. After completion of 10^{th} class the students are expected to achieve minimum competencies of arithmetic geometry and mensuration and data handling. Student is about to continue his education of his desired choice. To continue that education the required skills of mathematics should be developed.

Every student has to treat the classroom mathematics is same as the daily life mathematics. He should be able to apply the mathematics learnt in the classroom in his daily life.

To achieve above standards in mathematics, the teachers, the classroom, the textbook and the evaluation should be as following :

a. Classroom Environment – Teaching Learning Processes

The classroom environment and the organization of teaching learning activities in classes 9 and 10 should be on the lines discussed hereunder.

- The teaching learning activities at secondary stage should primarily develop mathematical reasoning in children .
- Opportunities should be given to promote abstract thinking.
- The teacher should explain secondary stage mathematical concepts linking them with real life situations and their use in day to day life. The teaching of mathematics should enable the children to understand how and why mathematics is around them every day, every time.
- The teaching learning activities should be organized giving priority to mathematical constructions and proofs.
- Priority should be given to mathematical concepts over formulas and definitions.
- Instead of solving problems using the formulas memorized, the children should be given opportunities to discover/invent formulas by generalization and logical reasoning.
- The teacher should show various ways of solving a problem. She should also encourage the children to solve problems in more than one way.
- The teacher should ensure conducive environment in the classroom that enable not just a few but all the children in the class to learn mathematics.
- Opportunities should be given to the children to create new problems and solve them.
- Before starting a new topic, the teacher should give information to the children about the mathematicians who worked in the respective field.
- The children should be given enough time to finish when they are given a problem/proof/construction. The teacher should go round the class observing the way the children approach the task and give suitable suggestions wherever necessary.

- Since analytical geometry and trigonometry are introduced for the first time at this stage, the teacher should inform the children about their importance and their relationships with other branches. She should also tell them the importance of these two in learning mathematics at this stage.
- The teacher should see that the children do the exercises in their notebooks and check them regularly.
- After understanding the concept, a model problem solving should be discussed on black board.
- The exercises of "Do this", "Try this" and "Think & discuss" should be practiced by students.
- "Do this" should be solved individually by students.
- "Try this" should be solved individually or in groups.
- "Think & discuss" should be answered after discussion in groups.
- The problems in exercises should be given as homework. Everyday, the solved home work problems should be corrected. And misconceptions of students should be corrected.
- The classroom environment should be conducive for the children to talk freely about their difficulties in learning mathematics and to clear their doubts.
- The teaching learning processes should link the formulaic calculation of areas of a triangle, a square, a circle, etc., with geometrical solutions.
- The teacher should see that the children solve problems in their notebooks. She should check them and give suitable suggestions regularly.
- The teacher should see that the children create new problems and solve them.

b. Teaching Learning Materials:

While teaching mathematics to classes 9 and 10, the teacher should keep the following materials readily available and use them definitely.

- Reference books and other books related to mathematics
- Magazines
- Graph paper
- Mathematical instruments box
- Geometrical shapes (2D, 3D)
- Modern electronic gadgets like TV, DVD, Tape Recorder, CDs, Computer, Internet, etc.
- Geo-board, etc.

c. Textbooks:

- The language used in the textbook should be so simple and easy to understand that the children can read and understand the books on their own. Especially, the language in word problems should not pose any problems to the children. The language should not stand in the way of solving a problem.
- Every concept should be introduced with the help of appropriate pictures and situations.
- The teacher should link the present concepts to the concepts already learnt by the children.
- The exercises in the textbook should make the children think and develop logical reasoning.
- Multiple solutions to the problems should be given in the textbook.
- The proofs should be given in an organized manner so that the children understand the logical reasoning embedded in them.
- Wherever necessary, historical background of the topic and information about the mathematicians who worked in that area should be given.
- Textbooks should give time and space to develop abstract thinking and mathematical reasoning. They should be so designed that learning proceeds from specific to general and vice versa.
- The textbooks should not give undue importance to definitions and formulas. They should facilitate understanding of mathematical concepts through various examples, proofs, and generalization. They should enable the children to devise formulas and give definitions on their own.
- The textbooks should be so designed that the children are not restricted to them but go beyond and consult relevant reference books to have a better understanding of mathematical concepts.
- The textbook design should promote learning mathematics by experimentation.
- The textbook should contain the aims of the book, organization of the book, directions / instructions to the teachers, and topic-wise (or unit-wise) competencies to be achieved.

d. Teacher Readiness – performance indicators

- The teacher should develop complete understanding of classes 9 and 10 textbooks.
- The teacher should prepare annual, unit, and period plans.
- The teacher should be prepared with all necessary teaching learning materials.
- The teacher should attend in-service teacher training programmes and use that knowledge/skills in classroom teaching.
- The teacher should complete the syllabus on time.
- The teacher should give exercises other than those in the textbook.
- The teacher should prepare question papers on her own and conduct the examinations.
- The teacher should see that all children achieve all competencies and all mathematical skills in each unit.
- The teacher should make notes in her diary about the progress of the children and the classroom management.

- The teacher should see that the children work on a project, exhibit it and discuss it in the classroom.This can be on the history of mathematical concepts or mathematicians, mathematical proofs, constructions, etc.

e. Evaluation

- As per RTE-2009 ACT, the CCE is implemented till elementary level (1 to 8th)
- The nature and qualitative items should be continued even in 9th & 10th class.
- CCE can be implemented in 9th class and 10th Class.
- The students in 10th class has to appear for a board exam in 10th class.
- The evaluation in the 10th class should not completely dependent upon the marks in the annual test. But there should be importance for the internal assessment during teaching learning process.
- Projects in mathematics, assignments, portfolios of children etc can be taken into consideration for internal assessment.
- As this internal assessment conducted regularly, it identifies of the difficulties of the learner in learning mathematics. Therefore the stress and strain caused by board examinations can be reduced.
- The teacher should prepare the question paper herself and conduct the Summative Evaluation.

Implications of critical pedagogy:

- ***Critical pedagogy is a strategy for construction of knowledge.***
- ***Reflective thinking skills, critical thinking skills, dialectical thinking skills and creative skills play an important role in critical pedagogy. The TLP should be organized to promote these skills. In all subjects it should be applied.***
- ***Teacher should know that students have life experiences and their own knowledge that is key in shaping their education and learning.***
- ***Good Education System or school does not blame students for their failures or strip students of the knowledge they bring to the classroom.***
- ***A deep respect should exist between teacher and student.***
- ***We should think in terms of teacher student and student teacher that is "A teacher also learns and a learner who teaches".***
- ***The professionalism of the teacher might be respected.***
- ***It is vital to know children's culture, knowledge base, language etc.***
- ***Part of the role of any educator involves becoming scholar and a researcher.***
- ***Praxis is a problems solving method that is to identify a problem, research the problem, develop a plan of collective plan of action, implementation and evaluation.***

\- ***APSCF-2011***

11. Resources for Teaching mathematics

Introduction

Remarkable progress in the information and communication technologies (both hardware and software) in the past decade or so has made it possible for teachers to access quality digital resources and use them either directly or indirectly in the classroom.
What makes it attractive for teachers are some unique features of these resources such as:

1. Extensive availability
2. Range of resources across topics and grades
3. Innovative ideas
4. Can be used on or offline
5. Resources in Mathematics from other countries / regions can easily be used / customized for our schools as math is universal subject
6. A large number of non-text resources like audio / video / applets etc.
7. Very low recurring cost

Teachers have taken to this new idea with enthusiasm (is it because of novelty alone? quite possible). Of course, there are critics who do not approve their usage. There are several studies to understand the effective of using these resources in the classroom. Some of the data has provided useful insights and pointers.

Digital resources can surely augment meaningful learning experiences of the child can be specifically used for the following:

1. Introduction of to a new topic / Conceptual understanding
2. Formation of key concepts
3. Repetitive practice (read drill)
4. Self relearning
5. Group work
6. to develop mathematical reasoning, communication and problem solving abilities
7. To develop creative abilities among the children as well as teachers

Teachers can use them in their classes if they fit into their lesson plan.
The following list gives a rough idea of the range of digital resources (mostly free) available on the internet. This is only indicative and by no means representative on comprehensive to be used more as pointers - and the readers are requested to make their own judgments regarding their suitability to their work.
The resources have been grouped into a few (loose) categories to facilitate easy navigation and exploration.

Websites

General

The mathforum@ Drexel University (http://www.mathforum.org)
The Centre for Innovation in Mathematics Teaching (CIMT) (http://www.cimt.plymouth.ac.uk)
Math cats – Fun math for kids (http://www.mathcats.com), count on (http://www.counton.org)
1. Illuminations - Resources for teaching maths (http://illuminations.nctm.org) Interactive (http://www.shodor.org/interactivate)
Gadsen Mathematics Initiative (http://www2.gisd.k12.nm.us/GMIWebsite/ImathResources.html)
2. Mathematical Interactivities – Puzzles, games and other online educational resources (http://mathematics.hellam.net)
3. National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html)
4. Mathnet – Interactive mathematics in education (http://www.mathsnet.net)
NewZealand maths (http://www.nzmaths.co.nz)
The Mactutor History of Mathematics archive (http://www.history.mcs.st-and.ac.uk/history)
Math cartons (http://www.trottermath.net/humor/cartoons.html)
Math Comis (http://home.adelphi.edu/~stemkoski/mathematrix/comics.html)
Mathematical quotation server (http://math.furman.edu/~mwoodard/mqs/mquots.html)
Wolfram Mathword – The web's most extensive mathematical resource (http://mathworld.wolfram.com)
Optical illusions and visual phenomena (http://www.michaelbach.de/ot)
Optical illusions gallery (http://www.unoriginal.co.uk/optical5.html)
Teachers resources online (http://www.cleavebooks.co.uk/trol/index.html)
Interactive : Activities (http://www.shodor.org/interactive/activities/#fun)
Maths articles (http://www.mathgoodies.com/articles)
Math words and some other words of interest (http://www.pballew.net/etyindex.html)
Portraits of scientists and mathematicians
(http://www.sil.si.edu/digitalcollections/hst/scientific-identity/CF/display_results.cfm?alpha_sort=R)
Let epsilon < 0 (http://epsilon.komplexify.com)
Grand illusion (http://www.grand-illusions.com)
Portrait gallery – Mathematicians (http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeid=2437&bodyId=2241)
Maths teaching ideas (http://www.teachingideas.co.uk/maths/contents.html)

E-books

Illustrated maths formulas – salim (http://www.arvindguptatoys.com/arvindgupta/mathformulas.pdf)

Ramanujan – the man behind the mathematician Sundaresan and Padmavijayam (http://gyanpedia.in/tft/Resources/books/ramanujan.doc)

A mathematician's apology – G.H.Hardy (http://math.boisestate.edu/~holems/holmes/A%20Mathematician%27s%20Apology.pdf)

Puzzle maths – G.Gamov and stern (http://www.arvindguptatoys.com/arvindgupta/puzzlemath.pdf)

1000 uses of a hundred square – Leah Mildred Beardsley (http://www.mediafire.com/download.php?detnojrueje)

Geometry comic book – Jeane Pierre Petit (http://www.mediafire.com/?ud0nnnujzyy)

Elements – Eucid (http://www.mediafire.com/?ud0nnnujzyy)

How children learn mathematics (http://gyanpedia.in/tft/Resources/books/mathsliebeck.pdf)

Suggested experiments in school mathematics – J.N.Kapur (http://www.arvindguptatoys.com/arvindgupta/jnkapur.pdf)

Primary resources – Maths (http://www.primaryresources.co.uk/maths/maths.html)

Proteacher! Maths lesson plans for elementary school teaches (http://www.proteacher.com/100000.html)

Maths activities (http://www.trottermath.net/contents.html)

Maths powerpoints (http://www.worldofteaching.com/mathspowerpoints.html)

Maths is fun – maths resources (http://www,mathsisfun.com)

Middle school portal for maths and science teachers (http://www.msteacher.org/math)

Maths games, maths puzzles and maths lessons designed for kids and fun (http://www.coolmath4kids.com)

Numbers

Magic, squares, magic stars & other patterns (http://recmath.org/Magic%20squares)

Number recreations (http://www.shyamsundergupta.com)

Broken calculator – Maths investigation (http://www.woodlands-junior.kent.sch.uk/mahts/broken-calculator/index.html)

Calculator chaos (http://www.mathpalyground.com/Calculator Chaos.html)

Primary school numeracy (http://durham.schooljotter.com/coxhoe/Curriculam+Links/Numeracy)

Quarks to Quasars, powers of 10 (http://www.wordwizz.com/pwrsof10.html)

Algebra

Algebra puzzle (http://www.mathplayground.com/Algebra_Puzzle.html)

Algebra tiles (http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/MathBitss07ImpFree.html)

(http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/MathBitss07ImpFree.html)

Geometry (http://www.cyffredin.co.uk)

The Fractory : An interactive tool for creating and exploring fractals (http://library.thinkquest.org/3288/fractals.html)

Tessellate (http://www.shodor.org/interactivate/activities/Tessellate)

MathSphere – Free graph paper (http://www.mathsphere.co.uk/resources/MathSphereFreeGraphPaper.html)

Paper models of polyhedral (http://www.korthalsaltes.com)

Problem solving

Mathpuzzle (http://www.mathpuzzle.com)

Puzzling world of polynedral dissections (http://www.johnrausch.com/PuzzlingWorld?contents.html)

Interactive mathematics miscellany and Puzzles (http://www.cut-the-knot.org)

Puzzles and projects (http://www.delphiforfun.org/Programs/Indices/projectsIndex.html)

10ticks daily puzzle page (http://www.10ticks.co.uk/s_dailyPuzzle.aspx)

Archimedes laboratory – teachers' resource: Improve problem solving skills (http://www.archimedes-lab.org/index_teachers.html)

Brain teasers (http://www.pedagonet.com/brain/brainers.html)

Gymnasium for Brain (http://www.gymnasiumforbrain.com)

Puzzles and games (www.thinks.com)

Miscellaneous

Mathematical imagery (http://www.josleys.com)

జనరల్

The mathforum@ Drexel University (http://www.mathforum.org)

సెంటర్ ఫర్ ఇన్నోవేషన్ ఇన్ మాథ్‌మాటిక్స్ టీచింగ్ (సిఐఎంటి) (http://www.cimt.plymouth.ac.uk)

పిల్లలకు సరదా గణితం – మాథ్ క్యాట్ (http://www.mathcats.com), కౌంట్ ఆన్ (http://www.counton.org)

ఇల్యూమినేషన్స్ గణిత బోధనకు ఉపకరించే రిసోర్సులు (http://illuminations.nctm.org) పరస్పర కృత్యం (http://www.shodor.org/interactivate)

గాడ్సన్ మాథ్‌మేటిక్స్ ఇనిషియేటవ్ (http://www2.gisd.k12.nm.us/GMIWebsite/ImathResources.html)

గణిత పరస్పరకృత్యాలూ పజిల్స్, ఆటలు, ఇతరాలు విద్యకు సంబంధించిన ఆన్‌లైన్ రిసోర్స్‌లు (http://mathematics.hellam.net)

మాథ్‌నెట్ విద్యలో పరస్పర గణిత కృత్యాలు (http://www.mathsnet.net)

నేషనల్ లైబ్రరీ ఆఫ్ విర్చువల్ మానిప్యులేటివ్స్ (http://nlvm.usu.edu/en/nav/vlibrary.html)

న్యూజీలాండ్ మాథ్స్ (http://www.nzmaths.co.nz)

ప్రాథమిక వనరులు గణితం (http://www.primaryresources .co.uk/maths/maths.html.)

ప్రాథమిక పాఠశాల ఉపాధ్యాయులకు గణిత బోధన ప్రణాళికలు (http://www.oroteacher.com/100000.html)

గణిత కృత్యాలు (http://www.trottermath.net/contents.html)

గణిత పవర్‌పాయింట్స్ (http://www.worldfteaching.com/mathspowerpoints.htm)

సరదా గణితం – గణిత వనరులు (http://www.mathsisfun.com)

గణజత, సైన్స్ ఉపాధ్యాయులకు పోర్టల్ (http://www.msteacher.org/math)

గణిత ఆటలు, పజిల్స్, పాఠాలు పిల్లలకోసం రూపొందించినవి (http://www.coolmath4kids.com)

అంకెలు

మేజిక్ స్క్వర్లు, మేజిక్ స్టోర్స్, ఇతర పద్ధతులు (http://www.recmath.org/Magic%20Squares)

అంకెలతో సరదాలు (http://www.shyamsundergupta.com)

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Some Important Readings in Mathematical Education

Pedagogy of Mathematics

– *Hriday Kant Dewan*

Before we begin certain issues entwined with the word "Pedagogy" need to be pointed out and elaborated. This word is commonly used as a convenient hold all but because of this and in some contexts inspite of this it still can not be completely discussed by itself. to explore its implications some other key elements need to be specified.

Can pedagogy stand by itself?

The first pre-requisite is the need to know the discipline being considered well. We need to know what it contains and its nature. this means to thinik about pedagogy of Mathematics is, what it includes, how it functions and then go to other questions. The first level answer to what it contains is : aritmetic and its generalization (i.e. algebra), geometry, statistics, analysis of number system and other such categories. It can be described as abstracting, organizing and generalizing of human experience related to quantity, shape and their transformation. Subsequently it becomes the basic language for building abstract and general ideas in all disciplines. Knowledge and constructs in Mathematics have gone far beyond the initial need of the human society for quantification, measurement and spatial visualization. As an abstract language, it links ideas and concepts in different domains. As it has grown, it has also sought to nurture commonalities across different domains of human experience.

The second pre-requisite is the need to articulate within Mathematics what we going to transact. The manner in which tables can be memorized is different from the way in which students can be helped to understand how to solve word problems or understand the idea of a variable. Pedagogy is not an epistemic category and cannot help you choose what you want to transact even though it may relate to and even be governed by these choices sometimes and vice verse. This relationship, where it can be seen, is striking and crucial. For example, you can not help children rote learn tables in a so-called constructivist manner nor have children explore open ended patterns in a classical behaviorist framework.

How do we construct what is to be transacted?

The multifaceted linkages of mathematics and its abstract nature prompt the NCF to suggest mathematization of child's understanding as a key goal for Mathematics teaching.

> ***"The Mathematics syllabus for the elementary classes has to revolve around understanding and using numbers and the system of numbers, understanding comparisons and quantifying them, understanding shapes and spatial relations, handling data etc.***

This means there needs to be an attempt to help the child abstract logically formulated general arguments, go into organizing her experiences deeply and equip the child to transend individual events and chance occurences. In a sense move towards a more general and rational view point. The Mathematics syllabus for the elementary classes has to revolve around understanding and using numbers and the system of numbers, understanding comparisons and quantifying them, understanding shapes and apatial relations, handlind data etc. In order to understand what aspects of these we need to transact and how we would transact it, the area aof Mathematics needs to be understood in a wider perspective. We need to have a broad picture and the entire scope in our minds. This would then need to be narrowed to the classroom and specific choices. For clarity on these we would require a statement in our mind about the reason for these choices.

The capability of solving problems can be considered in manyh ways. One very obvious way is to get the child to almost copy solutions. The problems given follow the examples. There are no other types of questions leave alone finding ways to address them. A good problem solving task requires being able to locate and find a variety of clues within the problem, find the formulation to solve it and then fulfill the steps. The expectation is development of the ability to solve just one kind of problem in the same way. The way Math is described here is not a system of handling operations but rather the ability to construct and understand algorithms.

This logically leads us to the other question "Why do we teach Mathematics?" If children fall to learn to abstract and are not able to follow the logic, do we really need to teach these aspects of the subject to them? Is there a cultural bias to Mathematics or can there even be a genetic bias that implies only some children can learn it? Is abstraction in Mathematics, Science, Philosophy, other subjects including History and Music a non-univerrsal ability? Or is ther someting peculiar about abstraction in Mathematics? We can all enjoy the rhythm of a beat but to appreciate what is known as classical music or classical dance requires an experience or situations that do not appear to be universally available. Is the ability to generalize and play with numebrs and space similar?

In this situation what then should constitute the universal elementary or secondary school curriculum? What is it that we can expect and want all children to learn such that they do not end up thinking of themselves or being described as incapable? The question asked can be, is it not sufficient for them to know counting numebrs and operations on them and a bit of decimal fractions and commonly used fractional numbers? Do we need to insist on making Math abstract and apparently so complex that many cannot follow it? Is the fact that children do not understand a certain kind of Math and are terrorized by it, a result of the way it is taught or is it due to the kind of content covered? Is terror the nature of the subject itself? So there is a complex interplay bewteen the questions - What is Mathematics? And what area of Mathematics is needed and can be transacted in elementary classes? In this we need to also consider whether all of it need to be universally learnt at this stage. We have to spell out (a) why is it needed for that age group, that backgound and in that historical context for children and (b) can it be learnt by students of that backgound at that stage given the circumstances of schools and teachers. The choices made need to be able to go through these filters. It is obvious that it is neither easy to construct these fillers with comprehensive information and arguments and nor is it easy to reach a consensus on implementing and discussing them given the hiatus these abilities seem to provide in the social and economic status accessibiligy.

As is evident from above content, 'what is pedagogy?' is difficult to address on its own. Its scope and conceerns are not articulated very precisely and there is not enough consensus on how it may be defined. There is, however, a common sense understanding that guides the way it is used generally.

What is Pedagogy?

Pedagogy is broadly used to imply the way a subject will be transacted. Described thus there are many obvious components of the word pedagogy. They include classroom transaction and process, nature and type of teaching-learning materials, assessment system, teacher student relationship, the nature of student engagement, the classroom arrangement etc. This is of course influenced by (and for some people includes) the chosen set of content, information, skills and concepts to be transacted and acquired. Pedagogy needs to worry about the inclusino of all the learners in the learning engagment. This implies the need for an awareness and sensitivity towards diversity and a concern about choices and context inthe syllabus. If you carefully consider the mainfestations of pedagogy in the classroom, then we know that it is also concerned with the way teachers are prepared, how they are dealt with administratively, the school building, the classroom, the social, economic and political undercurrents existing due to the diversity in the classroom and among teachers. There may also be other systemic and contextual aspects that may influence how transaction takes place. This then becomes a really extended set.

We would here, limit ourselves to some of the aspects. In these a few of the clearly discernible aspects mentioned above will be reiterated as issues that critically influence pedagogic consideration. These include :

a) Aims of teaching Math.

b) Nature of Mathematics and its key principles.

c) The teacher and her perspective.

d) How children learn Mathematics.

e) The attitude to the subject in society.

This will help us derive specific expectations and purposes for different class and age groups. This is what consitutes the syllabus. The first two components have to be informed by the so-called subject, its nature, purpose for human society and for the students for whom the transaction program is being developed. One has to keep in mind the person who is going to transact the learning so as to understand what the aims, expectations and learner backgounds demand from her. The third is: is there any specific understanding that we need about how this subject is learnt? This will help us construct classrooms that aid learning. The fourth is the prevalent attitude in society about Math-be it teaches, students or parents. All these contribute critically to the pedagogy of the subject.

Teaching Mathematics : Some Approaches

Discussing teaching-learning of any subject requires a basic understanding of now children learn. That should form the basis of our program particularly if each different component of the subject has a character that gives a specific tinge to its learning. The experience of these components for a particular child and the nature of the expectations from her can also be very different in comparison to the other children. For many years, Mathematics learning, like all other learning was considered to be linear and through repeated practice. Whatever was to be learnt had to be broken up into small components and given to children to practice bit by bit. The MLL (Minimum Learning Level) was a crucial example of this approach. In this the pedagogy was claimed to be competence directed.

There is also an expectation from the text book and other materials that for each small element termed as 'competency', there would be one page or one section entirely devoted to it. It was expected that once the child has gone through this she would automatically and surely have developed that once the child has gone through this she would automatically and surely have developed that part of the competency and needs now to go on to learn the next part. The MLL document itself used the word competency in many different ways. It was used loosely to describe information recall, procedure following, applying formula and in some cases concepts and problem solving as well. As a result of this, it is not clear how the word competency in the MLL document should be unpacked. The on-ground discourse on competency has also not moved forward. In this case Mathematics given its analyzed in the same framework and conceptualized as bit by bit and through practice of procedures and remembering facts.

Another element that pedagogy is crucially dependent on is the presentation of the teaching learning material (workbook and text book) and what it expects the child to do and how it suggests the class be organized and assessment made. The material needs to be clear on whom it is addressed to and therefore what it should contain. If the material is for the child then it has to have approapriate spaces, font size, suitable illustrations designed for children and appropriate language.

The textbooks and Mathematics classrooms before the advent of MLL and after the advent of MLL have remained essentially similar due to the fact that students are still being asked to practice algorithms and learn to numerate quickly. Articulation by the child, inclusion of the language of the child and allowing the child to explore and create new approaches to engage with mathematic situations are still not expected and not even accepted in materials. They follow the "consider the given solved example and do some more", approach to Mathematics learning.

We may also point out that the mention of a specific competency to be acquired meant the earlier mixed exercises that at least exerted the mind of the child in someway, also got limited to practicising just one option. It was at this time recognition for design, need for illustrations and color in the books emerged so at least the books were different. The principles informing the illustrations, design and other aspects however did not include the need to create space for the child to actively engage her mind.

In the absence of clear articulation, word competency was focused on explanation and telling short-cuts and facts. The key words 'learning by doing' and 'competency', in the context of Mathematics were inadequately explored and insufficiently addressed. Addition was a mere operation and acquiring it was the capability of adding single digit, 2 and more digit numbers with no carry over and then with carry over as colums additions. In ths quest to make Math a doing subject, competency based fractional numbers became definitions and operations. The itemized view of Mathematical ideas implied the narrowing of space for the child to formulate and articulate her own ideas and logic.

Two views on how to teach Mathematics

In analyzing how Mathematics is taught there are two contrasting views under which programs can be classified. We see classrooms constructed as a combination of these in some proportion. One view is that if you have students practice a lot of sums using algorithms and shortcuts, they eventually start understanding how the algorithm works and may get a sense of why it works. In any case they learn the steps clearly and are able to use it in any context. The nature of questions would, however, be varied.

The other view is that learning Mathematics is about developing an understanding of how the subject is constructed, its basic elements and working out the logical steps that lead to the algorithm and short-cuts in some areas. The child here is expected to be able to develop multiple strategies for problems and also use the algorithms if she finds it appropriate. The argument would not be that this is the best algorithm and has to be learnt by everyone but choose if appropriate. Students can also know, discover and discuss the nature of shortcuts and apply them if they so desire.

There are many examples given for the need for having children learn more than just algorithms. The simplest is addition of two digit numbers and the evidence that every often children introduced to these mechanically, end up viewing them as adding two independent one digit numbers placed in different columns.

There is also a lack of appreciation of the fact that when we multiply any number by a 2 or 3 digit number, the product from the 'tens' digit is not placed directly under the product from the unit place number. It is shifted by putting a cross under the units place. For example :

$$\begin{array}{r} 17 \\ \times\ 23 \\ \hline 51 \\ 34\mathrm{x} \\ \hline \end{array}$$

we are not always asked to seek a reason for the shift. There are similar examples from division as well.

Some people argue that the concepts of carry over or borrowing require an understanding of place value and therefore, unless we have children develop reasonable capability in plece value they will not be able to do additions and subtractoins that require such steps. The essential point here is that the focus is on learning the structure of the subject and the concepts. Once that happens the applications would gradually be learnt by the students. So in these while the eventual goals may be agreed upon, the approach is strikingly different.

Concrete to abstract : What does it mean?

Another aspect of pedagogy is related to the role and nature of materials in the classroom. We generally believe that abstract concepts are acquired through a process of creating, experiencing and analyzing concrete situations. There has been an increasing stress on putting in more and more concrete materials in the Mathematics classrooms. The idea of so-called Math bad has been supported and advocated widely. The feeling is that children understand concepts through the experiences in Mathematics laboratory. This needs to be considered carefully.

It is evident that the idea of using concrete materials and contexts for helping children learn is important. These serve as a temporary model to represent abstract concepts For example 5 stones are a concrete model for 5 and so 5 chairs. A triangle as it can portray some key properties of the triangle. It must be recognized that these artifacts do not fully represent the concepts of 5 or the triangle. They are only scaffolds for us to communicate what these terms mean inthe initial stages. Gradually learners have to move away from these concrete scaffolds and be able to deal with mathematical entities as abstract ideas that do not lend themselves to concrete representations.

A quadrilateral is closed figure bounded by 4 straight lines. A line is a one dimension infinite string that has no thickness. The point is that an actual line and hence a quadrilateral cannot be represented by even a drawing on the board leave alone by a concrete representation. So while it is important to begin with concrete experiences, gradually the child must articulate using her own language and move on. Mathematics going through the stage of using pictures and then tally marks etc. has to transit to symbols. This is an essential component of learning to do Math. Teh learning of Mathematics has to culminate in being able to deal with mathematical ideas on their own without any scaffolds. Therefore, when we advocate the Math laboratory for senior schools there is both a pedagogic as well as an epistemic question about whether this is the appropriate direction to proceed in.

The idea of laboratory in Sciene is to have the students explore some phonomena. She would make observations related to it and then based on the observations attempt to deduce some kind of causal connections. Utilizing many such experiments and data from earlier experiences, the student can be checked by further experimentation. The epistemological touch stone for ideas in Science can be arguably experimental observations and validations. This unfortunately is not true for Mathematics and therefore using the Math lab to have children deduce or prove mathematical statements by measurements or through models, is an epistemic and also a pedagogic error. The attempt at this stage has to be to enable the child to deal with abstract ideas.

Unlike the rich experience of language that the child comes to school with, ideas of Mathematics are not so richly experience based. All children are able to deal with numbers and arithmetic that they need in daily life. They are also able to organize the space around them and carry out spatial transformation to the extent they need. This knowledge is profound and complex. It shows the innate capability of the child to acquire these ideas. All children in any society are able to deal with these ideas. The problem comes whien we attempt to transact Mathematics and want them to de-contextualize and arithmetic that they need in daily life. They are also able to organize the space around them and carry out spatial transformation to the extent they need. This knowledge is profound and complex. It shows the innate capability of the chhild to acquire these ideas. All children in any society are able to deal with these ideas. The problem comes when we attempt to transact Mathematics and want them to de-contextualize and abstract the number, shapes transformation, operations and why all these work. The discipline of Mth is to be able to talk aout abstractions and how relations between abstract quantities can be understood and developed. In the primary classes Social Science and Science are also largely experience based and there is recognition that abstract concepts should not be imposed at this stage. Even in the upper primary classes it is possible to make Science replete with concrete experiences and use the available experiences of the child as well as the experiences provided in the classroom to help her construct a frame work of concepts. Mathematics does not allow this easily.

A lot of Mathematics pedagogy depends upon how the teacher engages with children. The classroom atmosphere has to be such that children can participate, articulate their ideas, make mistakes and talk about them without fear. Such an Mathematics. There is no one method or one technique that we can recommend for teachers to follow. She has to follow the classroom and create processes that facilitate engagement and dialogue that move forward gradually but can also return to an earlier point and develop again in a different way. The key aspect of Math classroom has to be the recognitin that chidlren will develop mathematical ideas and concepts though assimilation with their own previous ideas and experiences and modify them in the process of interactions. Each of us develop our own way of solving problems. It may require exposure to a lot of algorithm and methods but with an openness to create and examine more. They should be able to absorb available ideas and accommodate them in their conceptual framework. The models that anyone of us use or the artifacts a student constructs can help her understand the problem and develop a strategy but would not help everyone. They will be different for each of us.

You cannot help a person learn Mathematics by giving her short-cuts or imposing on her your way of solving problem. Your way may appear very simple, neat and elegant to you but that may not be so for her. We categorize and use ideas in our own ways and use steps that we can think of. It is doubly difficult task to understand the problem and then also discover the underlying logic of the process you have used to construct the solution. Mathematics will be learnt when the student will develop her own strategy, use the concepts and the algorithm inthe way she wants. This clearly implies that chilfen must have the opportunity to do lots of problems and solve them in many different ways.

We must expose the learner to these different varieties and develop not only the capacity to construct their own answer but also look and attempt to analyze and comprehend somebody else's answers. They need to be unafraid of making mistakes and confident of articulating their understanding. The implications in the classrooms are that children will work on their own, in groups make presentations on the solutions they have found and construct new problems as well as new generalizations. The classroom has to be such that the child is involved and engaged at each moment.

There has been a lot of talk about constructivism and teaching-learning processes. There have been arguments suggesting that teaching-learning process should be constructivist. This is sometimes interpreted to mean that children should be allowed to follow their own paths and decide what they want to do. It must be emphasized here that like the use of materials in Mathematics the space for the child to articulate her own understanding and building upon it needs to be interpreted in the context of an organized sharing of knowledge with the child and the nature of the discipline. Once the basis of deciding the Mathematics curriculum is arrived at then the classroom and the school has to help the child develop capability in the areas considered important. The teacher cannot ask children what should be done. At best she can construct options that are in conformity with the goals and objectives set out in the program for them to choose from. The notion of constructivism itself and its relationship to Mathematics teaching-learning needs to be explored and analyzed more carefully.

Assessment in Mathematics

An important part of any pedagogical statement is assessment. Whild there are general principles. The general key principles of assessment such as (a) the purpose of assessment (b) the participationof student in the assessment process (c) the mechanism of assessment (d) the way feedback would be provided to the child.

The manner in which assessment is done at present instills a feeling of fear and purposelessness for most children. Except for those few who are confident of doing well, the others usually want to get over it quickly and scrape through somehow. No one sees a relation between the examination, performance in examination and learning. In Mathematics examin ations, particularly, the nature of the tasks given and the manner in which theyare assessed lead to children being afraid of not just the examination but even the process of engaging with Mathematics. The entire assessment process in aimed to exhibit what the child does not konw rather than to discover what she knows. Concepts of formative, summative evaluation and other such terms do not spell out the prupose, importance and implications of gooe assessment processes. In recent years we have talked about continuous and comprehensive evaluation, no examination assessment and have argued for the teacher providing extra support to children who lag behing outside the class.

The revocation of the examination, the no-deterntion policy and the idea of outside the classroom support may appear to be conceptually nice but it is not operationally possible.

Education is a dialogue between school, teachers and the children. It this dialogue is not facilitated with trust, and openness is disallowed it would result in serious distortions in the clasroom processes. In Mathematics specifically it is important for the child and the teacher to know what she knows and also havge a sense of areas that she is struggling with. The process of the child needs to be based on what she was able to do earlier. We need to grade the performance of the chiild in that period rather than grade her against other children. Assessment and expectation is an important part of the requirement to make an effort. The fear of examination cannot take away purpose that assessment serves.

The way society looks at Math is a combination of awe, fear and a passport to success. There are strong beliefs about those who are able to learn Mathematics being more intelligent and have a greater chance and capability to succeed in life.

Mathematics is looked upon as a filter that would separate those who would be moving towardws higher intellectual prusults and those who would take up less intellectual roles in society. The anxiety of occupyng the intellectual and technical roles leads parents and teachers to pur pressure on students to learn. There is sub-conscious beginning of sorting by declaring many students incapable of learning and therefore helping them by some short -cuts to pass the examinators.

The fear of assesement and subsequent doors that are assumed to open on learning Mathematics lead to a tense atmosphere in the classroom. The general feeling in the society that it is difficult and has to be such that it can only be done by a few, prevents any attempt to allow children to slowly develop their ability.

It is difficult to conclude this discussion but it is clear that in considering pedagogical aspects of Mathematics it is not merely methods, classroom arrangements and presentations styles that we are talking about. We have to comprehensively look at education and the context of Mathematics, children, parents and teachers along with their aspirations, to move forward on the understanding pedagory.

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The Nature of Mathematics and its Relation to School Education

— Amitabha Mukherjee

Background

Mathematics, among all school subjects, enjoys a unique - and paradoxical - status. On the one school education. It is taught as a compulsory subject right from Class I to Class X. Moreover, it is often regarded as a kind of touchstone : an educated person is one who knows Mathematics. On the other hand, it is the most dreaded of school subjects, leading to a widespread sense of fear and failure among children. Even adults who have gone through school successfully can be heard to declare : "I could never follow Math in school." (When some of us started the School Mathematics Project at the Centre for Science Education and Communication, Delhi University, in 1992, our aim was to address this fear. For a more recent articulation, see the Position Paper of the National Focus Group on the Teaching of Mathematics, URL http://www.ncert.nic.in/html/pdf/schoolcurrimulum/position_papers/Math.pdf)

The above dichotomy raises a number of questions. Some of these are : what is Mathematics and why should we teach it in school? Does the problem with school Mathematics have something to do with the nature of Mathematics, or the way it is taught, or both? Can everyone learn Mathematics up to a point? Waht Mathematics should we teach in school? How should we teach it?

To attempt to provide answers to all the above questions would be ambitious, even foolhardy. In this article I will focus on some changes that have taken place in the thinking about school Mathematics over the last five decades, and their impact as felt in India in the last few years.

Mathematics for all

Any contemporary discussion on school Mathematics must take into account the context of Universalisation of Elementary Education (UEE). Today, UEE seems to be an attainable target rather than a distant dream. The next milestone of Universal Secondary Education (USE) will surely form a major part of the educational agenda in the coming decade. Thus when we talk of school Mathematics we are talking of something that is addressed to all children.

Can everyone learn Mathematics? The answer, fifry years ago, would perhaps have been adults talk of children who 'will never be able to learn Mathematics'. How does this face up to the concerns of UEE/USE? Taking categorical position, the Position Paper mentioned earlier asserts that :

> *Our vision of excellent mathematical education is based on the twin premises that* ***all students can learn Mathematics and that all students need to learn Mathematics.*** *It is therefore imperative the we offer Mathematics education of the very highest quality to all children.*

The question which then arises is : what kind of Mathematics teaching can meet the needs of all students. To be able to address this, we need to achieve some clarity about the goals of Mathematics education.

> ***Given that all chidlren are going to be learning Mathematics up to Class VIII and perhaps Class X, the main aim of school Mathematics teaching cannot be to produce Mathematicians.***

The aim(s) of School Mathematics Education

Given that all children are going to be learning Mathematics up to Class VIII and perhaps Class x, the main aim of school Mathematics teaching cannot be to produce Mathematicians. It cannot, for that matter, be to help produce scientists and engineers, in spirte of the special and important place that Mathematics occupies with respect to these disciplines. What then are the goals of school Mathematics education? The Position Paper says : ***Simply stated, there is one main goal - the mathematisation of the child's thought processes.***

In other words, the aim is to learn to think about the world in the language of Mathematis, and to develop the kind of thinking that is special to Mathematics. On the other hand, a look at curricula and textbooks in force in the country would seem that 'university education', or perhaps 'IIT education', has dominated the content and style of school Mathematics. No wonder a majority of past and present school goers have no love for the subject!

What is Mathematics, anyway?

If mathematisation of thinking is the main goal of Mathematics education, we need to have some agreement on what contitutes Mathematics. If you ask people at random the question "What is Mathematics?" You will most likely get spontaneous answers "Addition, Subtraction, Multiplication and Division". (On second thoughts or if pressed, people usually add algebra and geometry.) Now these operations on numbers undoubtedly form an important part of Mathematics, but they alone cannot serve to difine Mathematics or mathematical thinking. I will not attempt to give a definition; instead, I give you some examples of mathematical thinking.

"The door is between me and the wall."

"There are around fifty toffees in the jar."

"This galss is tall but thin. It will take less water than the wide mug."

"Nineteen and fifteen is... twenty and one less than fifteen... that's thirty-four."

"The station is about fifteen minutes if you take the road, but there's a short cut which will get you there in ten minutees."

At first sight, it may seem that the first statement carries no evidence of mathematical thinking. For a pre-school child, however, articulating spatial relationships such as 'above', 'below', 'between', 'beyond' is an important part of mathematisation.

Mathematisation of thought is not an absolute, onetime event. Through school and beyond it, children and even adults continue to Mathematise. On the other hand, our curricula may contain a lot of things that students learn without any accompanying processes, and hence without contributing to the real learning of Mathematics. Here are some examples, which, unless backed up by appropriate classroom processes, could end up being learnt by rote.

"To divide someting by *m/n,* you multiply by *n/m.*"

"The LCM of *a* and *b* is *a* times *b* divided by the HCF of *a* and *b*."

"All triangles with the same base and height have the same area."

The problem of Abstraction

Youngchildren learn about the world by handling objects. Their introduction to Mathematics therefore, is through the same route. Yet Mathematics, even in Class-I, necessarily involves abstraction. Consider a statement from the lowest level of school Mathematics :

"Two and two make four."

This is a statement about two and four, which are abstract entities. The wheels of a bicycle, a pair of socks and two apples have something in common : a property which we can call 'two-ness'. "Two apples and another two apples taken together make four apples" is a statement about the physical world, which can actually be tested - unlike the above abstract statement.

Martin Hughes in his 1986 book "Children and Number" records many conversations with children, which show that children have a "surprisingly substantial knowledge about number "before they start school. However, this knowledge is not couched in the formal language of the Mathclassroom. A child may correctly count the number of bricks in a box, and predict that if there are eight bricks in it, two more bricks added will make ten bricks in all. Yet the same child has no clue when asked the abstract question : "How many is eight and two?"

Such experiments have subsequently been done by many others, with similar findings. The implication for the classroom is that activities with concrete objects should come before the transition to the formal, abstract language in which mathematical content is usually framed. Moreover, the transition from the informal to the formal should be specifically addressed in our classroom practices.

The Construction of Mathematical Knowledge

Since the basic objects of Mathematics are abstract, we may wonder if they have an existence which is objective and independent of the human mind, or if they are constructs of the mind. This is an issue which philosophers have been debating since at least the time of the philosopher - Mathematican Rene Descartes (1596-1650). Are numbers, for instance, 'out there', or do they exist only in our minds? The various positions on this are summarised, for example, by Bertrand Russell in his very readable little book "Introduction to Mathematical Philosophy". I will sidestep this discussion for the moment to consider a slightly different aspect of the issue, one which is more directly relevant to the classroom.

It is generally agreed now, following the work of Piaget, Vygotsky and others, that children do not acquire knowledge passively. Rather, each learner actively constructs knowledge for herself. The process of knowledge constcuction involves interacting with the external world as well as with other people. Thus it does not matter whether mathematical entities have an objetive existence or not : We all have to go through the process of constructing them for ourselves.

Although piaget was not really concerned with school Mathematics, his work bears directly on the learning of Mathematics at the early stages. Constance Kamil has argued, for example, that yound children do not discover arithmetic, they re-invent it. At first sight this may seem contrary to the claim that pre-school children have a substantial knowledge of Mathematics, or at least number. However, there is no real contradiction if we remember that children are exposed to many contexts for mathematical knowledge before they enter school.

Is Mathematical Knowledge Unique?

Before we turn to the implications of these considerations for the classroom, we have to address the issue of what Mathematics to teach. Should our curricular choices be dictated by the structure of mathematical knowledge alone? If so, is this structure unique and universal? If this question is posed to a professional Mathematician, the likely answer will be an emphatic YES. However, we must remimber that members of the Mathematis research community are a self-defined, closed social group. As argued earlier, the aim of school Mathematics education cannot be to secure for learners membership of this elite group.

Researchers in many countries, including India, have documented many different traditions in Mathematics. Some of these are found in tribal and other isolated communities, while others - labelled 'street Mathematics' - can be seen to co-exist with the formal Mathematics taught in schools. Masons, plumbers and other artisans are ofter found to use their own, trade-specific, forms of Mathematics.

At a deeper level, the kind of Mathematics that engages the community of mathematicians at any place and time is determined by the other social groups to which the mathematicians belong. Considerattions of race, language nationality and religion cannot be ruled out, even though mathematicians may like to believe they are above and beyond such influences. The picture of Mathematics as a subject that has evolved linearly, largely in the West, from Euclid through Newton to the present day, is one that is increasingly challenged these days.

Implications for the Pedagogy of Mathematics

The above considerations naturally lead to some conclusions on how Mathematics should be taught. Since this volume carries an article on the Pedagogy of Mathematics, I will be brief.

1. Children should be provided contexts in which the learning of Mathematics can take place. These contexts have to be 'realistic' but not necessary real.
2. In th early classes there should be plenty of opportunity for children to handle concrete objects.
3. Special attention should be paid to the transition to the formal, symbolic mode. Early teaching of algorighms is to be discouraged.
4. Learning basic skills is important, but thinking mathematically even more important.
5. Learners should not be given the impression the mathematical knowledge is a a finished product.
6. Overall, the teacher should play the role of a facitlitator with each learner engaged actively in the processes of learning Mathematics.

Conclusion

It may appear that issues related to the nature of Mathematics belong to the reaim of philosophy, and have little relevance to the teaching of Mathematics in elementry classes. However, as argued above, there is in fact a profound connection. It is important, therefore, for people involved in school Mathematics - teachers, school heads, teacher educations, etc. - to engage at some level with the king of issues discussed here. How best this can be done remains an open question.

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A Culture of Enjoying Mathematics

– Shashidhar Jagadeeshan

Introduction

It appears that whether we like it or not, Mathematics pervades all aspects of our lives. Whether you are a farmer or a techie, a comfortable relationship with Mathematics, and competency at the level at which one uses it, is a requisite in an equitable society. Some will argue that even if the content of Mathematics learnt at school is forgotten, students will retain the ability to think clearly and logically (an essential life skill) because of their exposure to mathematical reasoning. The tacit assumption here is that learning Mathematics will not only help us in our daily lives but will also enhance the quality of our life. How ironic that for a vast majority their experience with Mathematics is so contrary to this assumption. Enough has been written bemoaning the state of Mathematics education the world over, and the term 'Mathphobia' has become part of common parlance. A major reason for school dropout is the inability to cope with Mathematics; it seems to be a universal phenomenon that many students fear and dread Mathematics. Sadly, this ffeling often persists into adulthood.

There have been many attempts to reform Mathematics education, and huge sums of money have been dedicated to this cause. Unfortunately, the motives for reform are suspect and, in my opinion, this is part of the problem. Advanced nations want to improve their citizens' Mathematics competency out of a fear that citizens of rival nations are outperforming them. Emerging nations want to improve their Matheducation so that they can create a 'knowledge society'. Humans empowered with knowledge are seen as a great asset in the market place. Reforms based on these motivations do not seem to have made much impact in the long run on Mathematics education (although there was a brief 'golden age of basic science' in the US thanks to the Sputnik fear).

If we are to make any headway in addressing both problems, that of poor mathematical competency and that of Mathphobia, we need to explore several questions first. What is the nature of Mathematics and how do our particular biases impact curriculum design? What is the relationship that students and teachers share with Mathematics? Waht are the myths or beliefs that students and teachers have about Mathematics? And perhaps most importantly, what are the factors that motivate humans to learn? In this article I hope to begin such an exploration by first describing the various ways in which Mathematics is viewed and experienced, and how these views might affect curriculum if applied in isolation. I then go on to look at curriculum design and pedagogy and see if we can truly create a culture of enjoying Mathematics not just for a elite few but for all.

The Blind Men and Mathematics

We are all familar with the famous Jataka tale about the blind men and the elephant. Each one makes tactile contact with a different part of the elephant, and comes up with descriptions ranging from a wall to a rope! Mathematics too suffers from partial perceptions. Perhaps the mystery, depth and richness of Mathematics is revealed in the fact that it can be seen in so many different ways. Let us look at some of these perceptios and how they impact curriculum design and pedagogy.

Mathematics as accountancy

For a large majority of people, Mathematics is synonymous with accountancy. Perhaps it is not unreasonable to say that the bulk of humanity uses Mathematics to compare prices, make sure they are not being cheated of the correct change, perhaps calculate interests, discounts and rebates; some may even calculate areas and volumes. The more advanced may use it in book keeping. It is also true that many discoveries in arithmetic probably came from the need to keep records of land and accounts of trade. Examples that comes to mind are preliminary trigonometry and mensuration, motivated by the need to calculate land holdings in the Nile Valley. Perhaps the motivation for the discovery of the Hindi-Arabic numeral system came more from the need to do large calculations in astronomy than from the need to do book keeping. But surely this discovery, considered one of the 'greatest intellectual feats of humans, 'has had its major and number manipulation is synonymous with Mathematics.

If this is one's only experience wth Mathematics, then one will design the curriculum and teach Mathematics as if it were a science of algorithms to do mechanical calculations and lose many students in this drudgery. Sofia kovalevskaia expresses this wonderfully : "Many who have never had the occastion to discover more about Mathematics confuse it with arithmetic and consider it a dry and arid science. In reality, however, it is a science which demands the greatest imagination."

Mathematics as problem solving and mental gymnastics

One of the major features of Mathematics is problem solving, and many who discover the thrill of problem solving at a yound age become professional mathematicians when they grow up. However, if this aspect of Mathematics is distorted, and seen in the wrong perspective, it becomes a source of fear and aversion toward Mathematics. Since talent in problem solving appears at a very yound age, children are often classified as 'brilliant' or 'dull' based on this single ability. When an education system euqates a child's self worth with their mathematical problem solving ability, it does great harm both to the ones who are adept at problem solving, and to those who are not. Those who find problem solvign difficult, and who then go on to be labeled as 'stupid' (either by society or by themselves), develop a deep fear and aversion to all of Mathematics. This self image is often linked to their self esteem, leading to feelings of insecurity and shame. All of us have encountered perfect strangers who have a great need to confess how bad they are at Mathematics. On the other hand, those who are very adept at problem solvign and Mathematics are automatically labeled 'intelligent', and run the risk of becoming one-demensional human beings with poor social skills. I leave it as an easy exercise to name your favorite mathematician as an example to illustrate this point!

There is no doubt that a large part of mathematical theory is motivated by the desire to solve difficult problems. Fermat's Last Theorem is a famous example. However, not all mathematical problems are of the same order. Some problems are indeed very profound and like the tip of an iceberg, reveal deep aspects of Mathematics. Many problems (an endless plethora) are simply mental gymnastic often created by working backwards from solutions, requiring some inane trick to solve.

Those who find problem solving difficult, and whothen go on to be labeled as 'stupid' (either by society or by themselves), develop a deep fear and aversion to all of Mathematics. This self image is often linked to their self esteem, leading to feelings of insecurity and shame.

These problems form the core of most of our competitive exams and are used as a sieve to week out applicants. Any system that uses this gymnastic ability as a yardstick to decide how to distribute access to resources such as education and jobs will surely create a skewed society. The effects of these are already being seen today at our institutions of higher learning. Studetns who have been put through a grind of mindless problem solving are burnt out and have no motivation to learn anything new. Such students ahve a very narrow view of Mathematics and very few will choose Mathematics research and teaching as a career. I have heard senior professors and administrators bemoaning the fact that it is very hard to find competent people to teach Mathematics at many new prestigious institutions in India. Imagine the fate of the many thousands of students who have, after several years of preparation, failed to get access to so-called quality education. With damaged psyches and bruised self confidence, what kind of learning can take place? Further the erroneous identification our society has made between intelligence and mathematical ability has led to a dismal state of education for those interested in pursuing the humanities, because disproportionate funds are made available to science education. Many students with no real interest in the sciences and perhaps very gifted in other areas still pursue science.

Mathematics as the 'language of the universe' and as a useful tool in modern society

With Galieo, Mathematics has begun to be seen as the language of the universe. Those who seek to unravel the mysteries of the universe see Mathematics as a sixth sense needed to comprehend the universe. We marvel with the physicist Eugene Wigner who gave a lecture in 1959 titled 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences. WWigner ends his lecture by saying, "The miracle of the sppropriateness of the language of Mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be greateful for it and hope that it will remain valid in future research and that it will extend, for better or for warse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." For many who pursue the more theoretical aspects of science, it is this 'miraculous' aspect that they most appreciate in Mathematics.

Mathematics with us brilliant ability to model phenomenon has had a far reaching impact in all aspects of our lives and on many fields of study, ranging from biology to economics.

Mathematics with its brilliant ability to model phenomenon has had a far reaching impact in all aspects of our lives and on many fields of study, ranging from biology to economics. This modeling ability has also made Mathematics and extermely useful tool for a wide variety of people ranging from businessmen to engineers. The vast majority of us use computers these days with no clue as to how they work, and similarly Mathematics is used as a tool by practitioners who have no clue as to why the tool works. A view of Mathematics that demands that its utility be demonstrated at all times will also have an adverse effect on Mathematics curricula and the teaching of Mathematics. The trouble is, very little of school Mathematics can be shown to the student to be applicable in a real sense. Most often the examples are rather contrived and meaningless. Furthermore, an attitude that says, "I will learn something only because it is useful" comes in the way of true learning.

A utilitarian approach to Mathematics with benefits to be reaped in the furre, while doing mechnanical and meaningless calculations in the persent, is not going to inspire students. It makes it boring, and Mathematics loses its playful and joyful aspect. As Julian Williams put it "The average student needs emotional and intellectual satisfaction now, not just in five or ten years' time, when they become adults!"

Mathematics as truth and beauty

We now enter esoteric descriptions of Mathematics! All pure mathematicians worth their salt will declare that the reason that they do Mathematics is because it is beautiful. If they are Platonists then they will further declare that they are in search of 'mathematical trugh', something to be discovered rather than invented. for all pure mathematicians? "A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas, like the colours or words, must fit together in a harmonious way. Beauty is the first test : there is no permanent place in the world for ugly Mathematics."

In my opinion however, the strongest motivation for pursuring Mathematics is experienced at an errotional level. All mathematicians, no matter what their view on the nature of Mathematics, will agree that in the process of creating Mathematics they experience a sense of 'illumination spreading throughout the brain'. Alain Connes, a Fields medalist (the highest honour one can achieve in Mathematics), describes this sensation as follows: "But the moment illumination occurs, it engages the emotion in such a way that it's impossible to remain passive or indifferent. On those rare occasions when I've actually experienced it, I couldn't keep tears from coming to my eyes."

The view of Mathematics that it is akin to a creative art form, and that only those who have tasted the heady joy of discovering Mathematics truly understand it, has the strongest appeal to most people (including myself) who study Mathematics for its own sake, and not only for its applications or other aspects as discussed above. From time to time mathematicians have lamented the fact that it is because both teachers and students do not truly understand this nature of Mathematics that we have distorted its curriculum and teaching.

However, a view that says that all mathematical experiences that should be similar to experiences of art or music also has its limitations. Beauty in art and music is relatively easily accessible to most human being, but to see the beauty of Mathematics requires a special connection to it and a fair degree of training. A large part of school Mathematics often does not have a rich enough structure to reveal its beauty; in fact in these years it is problem solving that draws most children to Mathematics. If Mathematics is an art form, then why force all children to learn Mathematics? If we make Mathematics optional based on early reactins to it, are we being responsible to children? Since aesthetics is a matter of taste, must we then allow teachers to fashion their curricula according to their taste? This will surely not satisfy the taste of all their students, let alone help them use Mathematics as a foundation to learn other subjects or earn a livelihood.

Then there is the question of why society should support mathematical activity. Most artists need patrons or find buyers for their art work. Mathematicians do not sell their theorems for a living! Frankly it is because policy makers see Mathematics as a useful tool that most people in the field of Mathematics are able to feed themselves! They either teach Mathematics or 'do' Mathematics which is considered useful for a living. A tiny minority is supported for doing Mathematics for its own sake.

Mathematics for all?

If we insist that Mathematics be part of the core curriculum for all students then we must also make it a ffundamental right that all students enjoy learning Mathematics! A renowned Carnatic musician once told me that Carnatic music enjoyed a huge revival some years ago, thanks to the efforts of several yound musicians who created and nurtured a broad 'rasika' base. These young people revived the 'sabha culture' in Chennai and hundreds of other small towns and villages throughout South India. Musicians young and old, amateur and accomplished, now have an appreciative audience and can earn a decent livelihood.

Can we create a culture of enjoying Mathematics? Surely this is the only holistic solution to the problems discussed so far. This can happen only if all stake holders really get a feel for the joy and thrill of doing, using and learning Mathematics. This seems a utopian dream given the current state of affairs - unimaginative curricula, poor infrastructure, poorly prepared teachers ("Mathematicians are not interested in teaching children, and teachers are not interested in doing Mathematics," says Paul Lockhart) and a culture of fear and anxiety as far as Mathematics is concerned. But like all revolutions change must begin at both the individual / grassroots and the systemic level.

It we insist that Mathematics be part of the core currinulum for all students then we must also make it a fundamental right that all students enjoy learning Mathematics.

At a systemic level we need to delink mathematical ability from intelligence. We need to help each child discover what they really love, but also learn to love the things they do. we should stop using arcane mathematical problem solving skills as the main criterion for access to resources such as education and jobs. have to urgently develop a broader base for assessing the aptitude and skills of our young. I am not suggesting a watering down of standards, instead I am asking for a broad based system of evaluation which takes into account the multiple facets of human intelligence, the capacity to be accountable and the more elusive quality of being a sensitive and responsible human being. A radical shift in this area will wipe out cultural anxiety towards Mathematics.

At the level of curriculum, we need to be clear about our goals for Mathematics education. At the minimum we would like everyone to be competent at numeracy, have sufficient critical understanding of data gathering and presentation so that they do not buy into false propaganda, and have the reasoning ability to detect fallacious arguments. For a smaller number of people, the goal would be one of competence in the use of Mathematics as a tool; for an even smaller fraction, to create new Mathematics rarely are creative mathematicians the programmed outcome of a system-they turn to Mathematics in spite of any system, as they cannot but do Mathematics!).

The Mathematics curriculum framework outline in the NCF 2005 is an excellent document and goes a long way in living very clear guidelines. However, there is an urgent need for a think tank of mathematicians, teachers and educational psychologists to create material keeping these goals in mind. We need to teach numeracy in creative ways to that these skills are mastered and retained. Since they will be used in daily practical situations, these skills should be evaluated through projects and games that simulate elevant situations, rather than through stressful exams. Since Mathematics often builds on itself, concepts need to be revisited but in creative rather than repetitive ways. The whole curriculum should be infused with the philosophy that Mathematics is the 'science of pattern recognitioin.' We need also to pay special attention to how pattern recognition is assessed. I have had several students who would not appear good at conventional textbook Mathematics, but nevertheless have a very strong spatial sense and are adept at recognizing patterns and solving logical puzzles. Children should have sufficient experience with solving meaningful problems and experience the thrill of having and insight. There is no ready made material available in the market that reflects all these demands. As I have said earlier, it is urgent that we set aside resourecs to create or at least put together such material in a coherent manner and train teachers to use them effectively.

At the level of the classroom, it is extremely important that a teacher creates a trui learning space. For such a space to be created there has to be a relationship of trust and affection between the student and teacher. The teacher must really enjoy doing Mathematics so that his students feel inspired. More importantly, he must help them understand their own fears and resistance to learning, and enable them to take ownership for their own learning Twenty years of education at Center For Learning has taught us that all these are not romatntic pipe dreams, but very much within the realms of possiblility.

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Mathematics and the National Curriculum Framework

– *Indu Prasad*

Mathematics is one of the oldest fields of knowledge and study and has long been considered one of the central components of human thought. Some call it a science, others an art and some have even likened it to a language. It appears to have pieces of all three and yet is a category by itself.

According to the National Curriculum Framework (NCF) 2005, the main goal of Mathematics education in schools is the 'mathematisation' of a child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. While there are many ways of thinking, the kind of thinking one learns in Mathematics is an ability to handle abstractions and an approach to problem solving.

The NCF **envisions** school Mathematics as taking place in a situation where :

1. Children learn to enjoy Mathematics rather than fear it.
2. Children learn "imprtrant" Mathematics which is more than formulas and mechanical procedures.
3. Children see Mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
4. Children pose and solve meaningful problems.
5. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth of falsity of statements.
6. Children understand the basic structure of Mathematics : arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathhematics, all of which offer a methodology for abstraction, structuration and generalisation.
7. Teachers are expected to engage every child in class with the conviction that everyone can learn Mathematics.

On the other hand, the NCF also lists the **challenges** facing Mathematics education in our schools as :

1. A sense of fear and failure regarding Mathematics among a majority of children.
2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time.
3. Crude methods of assessment that encourage the perception of Mathematics as mechanical computation - problems, exercises, methods of evaluation are mechanical and repetitive with too much emphasis on computation.
4. Lack of teacher preparation and support in the teaching of Mathematics.
5. Structures of social discrimination that get reflected in Mathematics education often leading to stereotypes like 'boys are better at Mathematics than girls. However the difficulty is that computations become significanly harder, and it becomes that much more difficult to progress in arithmetic.

The NCF, therefore, **recommends :**

1. Shifting the focus of Mathematics education from achieving 'narrow' goals of mathematical content to 'higher' goals of creating mathematical learning environments, where processes like formal problem solving, use of heuristics, estimation and approximation, representation, reasoning and proof, making connections and mathematical comunication take precedence.

2. Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging Mathematician.
3. Changing modes of assessment to examine students' mathematisation abilities rather than procedural knowledge.
4. Enriching teaceher with a variety of mathematical resources.

A major focus of the NCF is on removing fear of Mathematics from children's minds. It speaks of liberating school Mathematics from the tyranny of the one right answer found by applying the one algorithm taught. The emphasis is on learning environments that invite participation, engage children, and offer a sence of success.

Methods of Learning

The NCF says that many general tactics of problem solving can be taught progressively during the different stages of school : abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess-and-verify exercises, is useful in many problem-solving contexts. Moreover, when children learn a variety of approaches (over time), their toolkit becomes richer, and they also learn which approach is the best. Children also need exposure to the use of heuristics or rules of thumb, rather than only believing that Mathematics is an 'exact science'. The estimation of quantities and approximating solutions is also an essential skill. Visualization and representation are skills that Mathematics can help to develop. Modelling situations using quantities, shapes and forms are the best use of Mathematics, mathematical concepts can be represented in multiple ways, and these representations can serve a variety of purposes in different contexts.

For example, a function may be represented in algebraic form or in the form of a graph. The representation 'p/q' can be used to denote a fraction as a part of the whole, but can also denote the quotient of two numbers, 'p' and 'q'. Learning this about fractions is as important, if not more, than learning the arithmetic of fractions. There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw graphs, they should also be encouraged to think of functional relationships in the sciences, including geology. Children need to appreciate the fact that Mathematics is an effective instrument in the study of science.

The importance of systematic reasoning in Mathematics cannot be over-emphasised, and is intimately tied to notions of aesthetics and elegance so dear to Mathematicians. Proof is important, but in addition to deductive proof, children should also learn when pictures and constructions provide proof. Proof is a process that convinces a skeptical adversary; School Mathematics should encourage proof as a systematic way of argumentation. The aim should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning.

The NCF aslo speaks of mathematical communication - that it is precise and employs unambiguous use of language and rigour in formulation, which are important characteristics of mathematical treatment. The use of jargon in Mathematics is deliberate, conscious and stylled. Mathematicians discuss what appropriate notation is since good notation is held in high esteem and believed to aid thought. As chilfen grow older, they should be taught to appreciate the significance of such conventions and their use. This would mean, for instance, that setting up of equations should get as much coverage as solving them.

Organization of the Curriculum

The NCF recommends the following for different stages of schooling :

1. Pre-Primary : At the pre-primary stage, all learning occurs through play rather than through didactic communication. Rather than the rote learning of number sequence, children need to learn and understand, in the context of small sets, the connection between word games and counting, and between counting and quantity. Making simple comparisons and classifications along one dimension at a time, and identifying shapes and symmetries, are appropriate skills to acquire at this stage. Encouraging children to use language to freely express one's thoughts and emotions, rather than in predetermined ways, is extermely important at this and at later stages.

2. Primary : Having children develop a positive attitude towards, and a liking for Mathematics at the primary stage is as important as developing cognitive skills and concepts. mathematical games, puszzles and stories help in developing a positive attitude and in making connections between Mathematics and everyday thinking. Besides numbers and number operations, due importance must be given to shapes, spatial understanding, patterns, measurement and datea handling. The curriculum must explicitly incorporate the progression that learners make from concrete to abstract while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections, and on the development of skills of language in communication and reasoning.
3. Upper Primary : Here, studentss get the first taste of the application of powerful abstract concepts that compress previous learning and experience. This enables them to revisit and consolidate basic concepts and skills learnt at the primary stage, which is essential from the point of view of achieving universal mathematical literacy. Students are introduced to algebraic notation and its use in solving problems and in generalisation, to the systematic study of space and shapes, and for cnsolidating their knowledge of measurement. Data handling. representation and interpretatio form a significant part of the ability to deal with information in general, which is an essential 'life skill'. The learning at this stage also offers an opportunity to enrich students' spatial reasoning and visualisation skills.
4. Secondary : Students now begin to perceive the structure of Mathematics as a discipline. They become familiar with the characteristics of mathematical communication : carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions, and proofs justifying propositions. These aspects are developed particularly in the area of geometry. Students develop their facility with algebra, which is important not only in the application of Mathematics, but also within Mathematics in providing justifications and proofs. At this stage, students integrate the many concepts and skills that they have learnt into a problem - solving ability. Mathematical modelling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy. Individual and group expolration of generalisation, making and proving conjectures are important at this stage, can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.
5. Higher Secondary : The aim of the Mathematics curriculum at this stage is to provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that often conficting demands of depth versus breadth needs to be made at this stage.

On **Assessment,** the NCF recommends that Board examinations be restructed, so that the minimum eligibility for a State certificate is numeracy, reducing the instance of failure in Mathematics. At the higher end, it is recommended that examinations be more challenging, evaluating conceptual understanding and competence.

The NCFs vision of excellent mathematical education is based on the twin premise that all students can learn Mathematics and that all students need to learn Mathematics. It is, therefore, imperative that Mathematics education of the very highest quality is offered to all children.

Problem Posing

1. If you know that 235 + 367 = 602, how much is 234 + 369? How did you find the answer?
2. Change any one digit in 5384. Did the number increase or decrease? By how much?

Source : NCF 2005

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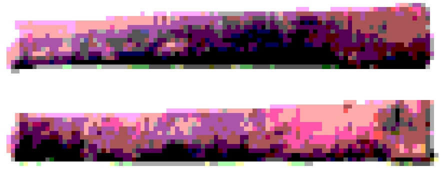
Number : The Role of Pattern and Play in its Teaching

— *Shailesh Shrali*

Pre-history

The concept of number is crucial to Mathematics, yet its origin may forever be hidden from us, for it goes far back in time. Human beings must have started long back to use the tally system for keeping records-livestock, trade, etc - but we may never know just when. A remarkable discovery made in 1960 in Belgian Congo of the Ishango bone, dated to 20,000 years BP, suggestes that the seeds of Mathematical thinking may go still further back than thought; for, carved on the bone are tally marks grouped in a deliberate manner, seemingly indicative of a Mathematical pattern (there is even a hint of a doubling sequence : 2, 4, 8). However, until further evidence is uncovered, the matter must remain as speculation. See the Wikipedia reference for more information.



Tally counting as a practice may well be as many as 50,000 years old; even today we use it to count, in various contexts, e.g. in a class election.

Base ten number system

The notation we use today - the base ten or decimal system - has its origins in ancient practices. Long back the Babylonians used a system based on powers of 60, and traces of that practice remain to this day - we still have 60 seconds in a minute, 60 minutes in an hour, 60 minutes in a degree (for angular measure). Later the Egyptians developed a system based on powers of 10, in which each power of ten from ten till amillion was represented by its own symbol. But this system differs from ours in a crucial way - it lacks a symbol for Zero.

A system of arithmetic without a symbol for zero sufferes from two difficulties. The first is that there is confusion between numbers like 23, which represents 2 tens and 3 units, and 203, which represents 2 hundreds and 3 units. Without the zerc symbol some way has to be found to indicate that the 2 means "2 hundreds" and not "2 tens". This can be done, but it is quite cumbersome. But a greater difficulty is that computations become significantly harder, and it becomes that much more difficult to progress in arithmetic.

The Greeks did not have a symbol for zero, and it is not surprising that the did not develop arithmetic and algebra the way the developed geometyr, which they took to great heights. It was in India that the symbhol for zero came into being (probably as early as the 5^{th} century), along with the rules for working with it. Not coincidentally, arithmetic and algebra grew in a very impressive manner in India, in the hands of Aryabhata, Brahmagupta, Mahavira, Bhaskaracharya-II, and many others.

On the other hand the ancient Indians did not progress anywhere as fat in their study of geometry. But it is striking that one area where the methods of algebra and analysis either into geometry in a natural way, namely trigonometry, did originate in India (in the work of Aryabhata, 5^{th} century AD).

"Concepts are caught, not taught". It is only by actual contact with collections of objects that concepts form in one's brain.

Abstraction and the number concept

Embedded in our brains is an extraordinary ability : the ability to form concepts; the ability to abstract common features and shared qualities from collections of objects or phenomena. It is this ability that lies behind the creation of language, and it is this that enables us to "invent" numbers. To understand what this means, think of a number, say 3. Is 3 a thing? Can it be located somewhere? No, it cannot; but our brains have the ability to see the quality of "threeness" in collections of objcts: three fingers, three birds, three kittens, three puppies, three people - the feature they share is the quality of threeness. This ability is intrinsic to the very structure of our brains. Were it not there, we would never be able to learn the concept of number (or any other such concept, because any concept is essentially an abstraction).

Even in something as simple as tally counting - creating a 1-1 correspondence between a set of objects ad a set of tally marks - our brains show an innate ability for abstraction : by willfully disregarding the particularities of the various objects and instead considering them as faceless entities.

Realization of this insight has pedagogical consquence; for, as has been wisely said, "Concepts are caught, not taught". It is only by actual contact with collections of objects that concepts form in one's brain. How exactly this happens is still not well understood, but I recall a comment wich goes back to Socrates (*the teacher's role is akin to that of a midwife who assists in delivery*).

The inverntion of algebra represents one more step up the ladeer of abstraction. To illustrate what this means, let us examine these number facts : 1+3=4, 3+5=8, 5+7=12, 7+9=16, 9+11=20. We see a clear pattern : the sum of two consecutive odd numbers is always a multiple of 4. This statement cannot be verified by listing all the possibilities, for there are too many of them - indeed, infinitely many. But we can use algebraic methods! We only have to translate the observation into the algebraic statement $(2n-1) + (2n+1)=4n$; this instantly proves the statement. Such is the power of algebra and also the power of abstraction - and this ability too is intrinsic to our brains.

Number patterns

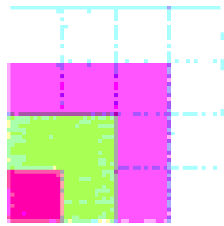
Another feature intrinsic to the brain is the desire and capacity for play. Most mammals seem to have it, as we see in the play patterns of their young ones - and what a pleasing sight it can be, to watch kittens or puppies or baby monkeys at play! But human beings have a further ability : that of bringing patterns into their play. When our love of play combines with the number concept and with our love of patterns, Mathematics is born. For Mathematics is essentially the science of pattern.

It is crucial to understand the element of play in Mathematics; for one is told, repeatedly, of the utility of Mathematics, how it plays a central role in so many areas of life, and how it is so important to one's career. But the element of play gets passed over in this view point; the subject becomes something one must know, compulsorily, and the stage is set for a long term fearful relationship with the subject.

From the earliest times - in Babylon, Greece, China, India-there has been a playful fascination with number patterns and geometrical shapes one can associate with numbers. From this are born number families - prime numbers, triangular numbers, square numbers, and so on.

Let us illustrate what the term "pattern" means in this context. We subdivide the counting numbers 1, 2, 3, 4, 5, 6, 7, 8, ... into two families, the odd numbers (1, 3, 5, 7, 9, 11, ...), and the even numbers (2, 4, 6, 8, 10, 12, ...). If we keep a running total of the odd numbers here is what we get : 1, 1+3=4, 1+3+5=9, 1+3+5+7=16, 1+3+5+7+9=25. Well! We have obtained the list of perfect squares!

There is a wonderful way we can show the connection between sums of consecutive odd numbers and the square numebrs; it is pleasing to behold and incisive in its power at the same time. All we have to do is examine the picture below : this property is closely related to one about

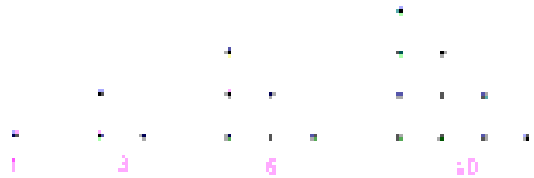


the trianglular numbers : the sequence 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ... formed by making a running total of the counting numbers : 1, 1+2=3, 1+2+3=6, 1+2+3+4=10, etc. They are so called because we can associate triangular shapes with these numbers.

There is just one red square; when we put in three green squares around it, they together make a 2 by 2 square; hence we have 1+3=2 times 2.

Pur in the fice purple squares and now you have a 3 by 3 square; hence 1+3+5=3 times 3.

Put in the seven blue squares, and now you have a 4 by 4 square; hence 1+3+5+7=4 times 4. And so on!



There are two striking properties that connect the traingular numbers with the square numbers (1, 4, 9, 16, etc), and consecutive triangular numbers is a square number; e.g., 1+3=4, 3+6=9, 6+10=16, ...; (II) if 1 is added to 8 times a triangular number we get a square e.g., (8 x 3)+1=25, (8 x 6)+1=49, (8 x 10)+1=81.

Why is there such a nice connection? A lovely question to ponder over, isn't it?

Here is another pattern. Take any triple of consecutive numbers; say 3, 4, 5. Square the middle number; we get 16. Multiply the outer two numbers with each other; we get 3 times 5 which is 15. Observe that 16-15=1; the two numbers obtained differ by 1. Try it with some other triple, say 7, 8, 9 : 8 squared is 64, 7 times 9 is 63, and 64-63=1; once again we get a difference of 1. Will this pattern continue? Yes, and it is easy to show it using algebra; but think of what pleasure discovering this can give a young child playing with numbers!

We find a similar but more complex pattern with the famous Fibonacci sequence, which goes 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...; here, each number after the first two is the sum of the preceding two numbers (e.g., 8=5+3). Repeat the computation with this sequence. With the triple 2, 3, 5 we get : 3 squared is 9, and 2 times 5 is 10; the squared number is 1 less than the product of the other two. With the triple 3, 5, 8 we get : 5 squared is 25, and 3 times 8 is 24; now the 8 is 64, and 5 times 13 is 65; once again the squared number is smaller by 1. And so it goes - a curious alternating pattern.

We see the same thing if we study collections of four consecutive Fibonacci numbers; for example. 1, 2, 3, 5. The product of the outer two numebrs is 5, and the product of the inner two is 67; they differ by 1. Take another such collection : 3, 5, 8, 13. The product of the ouer two is 39, that of the inner two is 40; once again, a differene of 1. And again the alternativn pattern continues. Astonishingly, even nature sees fit to use the Fibonacci numbers. If we keep records of the numbers f petals in various flowers, we find that the number is generally a Fibonacci number. Study the spirals in which pollen grains are arranged in the center of a sunflower; there are spirals running in clockwise and anticlockwise directions: you will find that the number of spirals of each kind is a Fibonacci number. Nature is just as fond of patterns as we are!

Many years back I used a textbook called "Pattern and Power of Mathematics". It is a nice title for a textbook, for patterns are what the subject is all about, and it is this that gives it its astonishing power. But - more important - it is this feature that makes us study the subject in the first place.

Large numbers, small numbers

There are numebrs, and then there are large numbers Children naturally like large numbers, and many of them discover on their own that there is no last number: however large a number one may quote, one only needs to add 1 to it to get a larger number. So the number world has no boundary! There are some who make a similar discovery at the other end - with small numbers; I recall a student telling me, many years back, how she could make an unending sequence of tinier and tinier fractions, simply by halving repeatedly; she could not believe that such tiny numbers could exist! She had made this wonderful discovery herself, and was very excited by it.

The ancient Indians loved large numbers, and here's a problem that shows this love. If I ask you to find a squared number that is twice another squared number, you would never succeed, because there aren't any such pairs of numbers. (Why? - there's a nice story behind that, but we cannot go into that now.) So we change the problem a little bit : I ask for a squared number which exceeds twice another squared number by 1. Now we find many solutions; e.g., 9 and 4 are squared numbers, and 9 - (2 x 4)=1. Here are some more solutions :

289 -(2 x 144) = 1,

9801 - (2 x 4900) = 1.

If we replace the word "twice" by "5 times" we find solutions to this too :

81 - (5 x 16) = 1,

(161 x 161) - (5 x 72 x 72) = 1,

and so on.

In the 7th centrury, Brahmagupta asked if we could find solutions with "5 times" replaced by "61 times". The smallest solution in this case is very large indeed - yet Brahmagupta found it :

(1766319049 x 1766319049 - (61 x 226153980 x 226153980) = 1.

Feel free to verify the relation.

I think the date is significant : the Indians were asking such questions thirteen centuries back! The love of play has been there in all human cltures, for a long time. There's no holding it back.

But now a strange thing happens. What began as play takes wing, and files away a mature subject, with an inner cohesiveness and structure that is strong enough to find application in the world of materials, living bodies, and finance - the "real world". Such flights have happened two dozen times or more in history, and no one really knows how and why they happen; but they do. Maybe it is God's gift to us. (But we do not always use it as intended; the power of Mathematical methods also finds application in the design of bombs and unclear submarines and other instruments of killing.)

Closing note

There are so many topics in which we can bring out the theme of pattern and play in Mathematics:

- Magic squares (arranging a given set of 9 numbers in a 3 by 3 array, or 16 numbers in a 4 by 4 array, so that the row sums, column sums, diagonal sums are all the same); not only do these bring out nice number relationships, but in the course of the study one learns about symmetry.
- Cryparithms (solving arithmetic problems in which digits have been substituted by letters; for example, ON + ON + ON + ON = GO; many simple but pleasing arithmetical insights emerge from the study of such problems);

- Digital patterns in the powers of 2 (list the nunits digits of the successive powers of 2; what do you notice? Now do the same with the powers of 3; what do you notice?)

These examples are woven around the theme of number, but the principle extends to geometry in an obvious way. Here we study topics like rangoli and kolam; paper folding; designs made with circles; and so on.

Alongside such activities, teachers could also raise questions relating to the role of Mathematics in society, for discussion with students and fellow teachers; e.g., questions relating to the use of Mathematics for destructive purposes, or more generally, "When is it appropriate to use Mathematics?"; or the question of why society would want to support mathematical activity. After all, most artists find partrons or buyers for their art work, but mathematicians do not sell theorems for a living! Is it that policy makers see field to sustain themselves, by teaching or doing useful Mathematics? The notion of usefulness takes us back to the question of appropriateness of usage. Such questions are not generally seen as fitting into a Mathematics class, but there is clearly a place for them in promoting a culture of discussion and inquiry.

We need not try to make a complete listing here - it is not possible, because it is too large a list, and ever on the increase. Instead, we with only to emphasize here that pattern and play are curcial to the teaching of Mathematics, for pedagogic as well as psychological reasons.

A great opportunity is lost when we make Mathematics into a heavy and serious subject reserved for the highly talented, and done under an atmosphere of heavy competition. It denies the experience of Mathematics to so many.

Suggested Reading

1. http://en.wikipedia.org/wiki/Ishango bone
2. "Number, The Language Of Science" Tobias Dantzig.
3. "The Number Sense : How the Mind Creates Mathematics" Stanislas Dehaene.
4. http://en.wikipedia.org/wiki/History of Mathematics

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Culture in the Learning of Mathematics

─ *K. Subramaniam*

At a recent workshop in Ahmedabad, we asked primary school teachers to talk about what their students do outside school, and whether it involves any Mathematics. The teachers spoke a lot. Their pupils, who came from poor urban homes, helped their parents sell vegetables. They made and sold kites, packetes bindi, agarbathis and many other things. They knew the price of vegetables for different units, knew how much profit they would make from selling a 'kori' (unit of 20) of kites. Kites had to be assembled from paper sold in packets and sticks sold in bundles - all in different units. Problems arose naturally whhile makin gdecisions about how much raw material to buy, how much to make and sell, how much time to spend, and so on. Children, together with older siblings or adults, were finding their own ways of getting around these problems. And all the time, they were dealing with numbers and Mathematics.

> ***Almost no school curriculum gives any place to such 'everyday' Mathematics. At best there may be an attempt to add some contextual details to enhance children's interest. Thus the Mathematics that children learn to do inside and outside school remain separate and disconnected.***

It was not the children telling us these things, it was the teachers. We asked them how they discovered that the children knew so much. They replied that when the children absent themselves from school, they visit their homes to find the reason. They talk to the parents and often find that the child was helping them ` perhaps hawking vegetables while the mother went on an errand. We were happy that the teachers took pains to ensure attendance, but we also felt a little uneasy with this reply. When we opened the worksheets prepared for the chidlren - this chain of schools used their own worksheets rather than a regular textbook - we did not find anything of what we had just heard about the children's lives. It struck us that teachers found out about the children's activities outside of the school, and not in the Mathematics classroom.

After some discussion with the teachers, we realized that they held strong beliefs about what counts as 'proper' Mathematics. A problem used in a Dutch study, 'If a plar bear weights 350 kg, about how many children weigh the same as a polar bear', was for them not a good problem, because it did not have all the data needed to solve it. The problems that the children were solving outside school often had incomplete data, did not have a precise single answer, and the children used not have a precise single answer, and the children used informal methods of solving them. So the teachers did not think that the children were really doing Mathematics. There seemed to be an invisible wall separating the Mathematics in school and the thinking and figurin gthat the children did in the context of economically productive activities.

This story is not an unusual one. In many poor urban households, children participate in economic activities. In a different social or geographical context, if one looks carefully, one will discover that here too children have opportunities to engage with Mathematics outside school. Almost no school curriculum gives any place to such 'everyday' Mathematics. At best there may be an attempt to add some contextual details to enhance children's interest. Thus the Mathematics that children learn to do inside and outside school remain separate and disconnected. of course, the larger issue here is of the relation between the school curriculum and life outside school. Since Mathematics is an abstract branch of knowledge, one may think that there is little to be said about its connection with culture and everyday life. Yet, many researchers have studied the relation between 'everyday' and school Mathematics leading to important insights.

Advocates of constructivism, following Piaget, stress the fact that children don't enter schools with empty minds waiting to be filled - they have already acquired complex knowledge in the domains that overlap with school Mathematics and science. Psychologists studying cognitive development have constructed a detailed picture of the spontaneous conceptions that children acquire. The first wave of contructivism was however criticised for focusing largely on individual learning. The criticism came from a broad range of perspectives that were more sensitive to the influences of culture and society. The implications of these critiques are still being worked out by researchers and thinkers in the Mathematics education community. Here we will look at some of the ideas and possibilities that have emerged from this debate.

The pioneering studies of street Mathematics by Terezinha Nunes and her colleagues, the anthropological studies by Geoffrey Saxe of the Mathematics of the Papua New Guinea communities, the studies by Farida Khan in the Indian context, and many other studies have revealed how Mathematics arises spontaneously in the context of everyday activity. These studies have also shown how everyday' Mathematics differs from school Mathematics. In everyday contexts, calculation is 'oral', and mostly uses additive build-up strategies. When an adult from the Mushari tribe in Bihar was asked to give the cost of ten melons if each melono costed Rs.35, he did not 'add a zero to the right' to straight away get 350. Instead, he first calculated the cost of 3 melons as Rs.105. Nine melons were Rs.315 and so ten melons were Rs.350. Exactly the same procedure was used to solve the same problem by a Brazilian child vendor in Nunes' study. The 'add zero to the right' strategy is a part of 'written' Mathematics, and is uncommon in everyday Mathematics. Proportion problems are usually solved in the everyday world through a build-up strategy rather than by using a 'unitary method' or the 'rule of three'. For example, consider the problem 'if 18 kg of catch yield 3 kg of shrimp after shelling, how much catch do you need for 2 kg of shelled shrimp?' A fisherman in Nunes' study calculated it as follows : we get 1 1/2 kg of shelled shrimp from 9 kg of catch, so 1/2 kg from 3 kg of catch. Nine plus three is twelve. So 12 kg of catch would give you 2 kg of shelled shrimp.

Since these procedures were oral, sometimes respondents forgot to complete a step of the calcultion, but the errors were usually small and the ansers reasonable. Nearly always, the calculation model was accurate. In contrast, school children often use the wrong operation for a problem and produce unreasonable answers. Culture and cognition seem to work together in everyday Mathematics to create a robust sense of appropriate modelling. When children are presented with a problem that they can understand, and are encouraged to find their own way of solving them, we see that their spontaneous solution procedures are often like those of everyday Mathematics. These findings have important implications for teaching and learning Mathematics. One can for example, re-conceptualize learning trajectories so that the problems, concepts and procedures of everyday Mathematics provide the springboard for more powerful mathematical concepts. The rich contexts that are familiar to children provide valuable scaffolding while solving a problem, verifying that its solution is reasonable and looking at a problem from different angles.

If we see cultural knowledge as merely a vehicle to deliver formal Mathematics that is otherwise 'difficult-to-swallow', then we may be adopting a view which is too narrow. We cannot simply mine what is present in the culture as a resource to push a particular curricular agenda. Putting cultural knowledge alongside formal knowledge leads us, as educators, to reflect more deeply about their relation. We need to not only take from the culture sources of mathematical thinking, but also give back to the culture waht it values hightly. In the long run, if a form of knowledge is to survive and flourish, it must have deep roots in the culture. We don't understand well the meeting points between disciplinary knowledge and knowledge that is dispersed as part of culture. Is such culturally dispersed knowledge incommensurable with the academic knowledge of the universities, as some thinkers in education have argued (Dowling, 1993)? Can the familiar dichotomies of folk vs formal knowledge, or traditional vs. modern knowledge capture the relationship between the two kinds of knowledge? In some domains of knowledge, cultural dispersion and transmission through formal institutions have both had a strong presence over long periods. A good example is classical Indian music. Another example is traditional medical knowledge, which is now reproduced through modern educational institutions. Both music and medicine as formal systems preserve a connection to the diversity of cultural forms - to popular music or to the many local and specific healing traditions. Much of the knowledge that we seek to impart in school has no comparable cultural presence or diversity of forms of expression.

Mathematics may have deep roots in our culture that we are still to become aware of. Among some members of the Mushari community, there is an impressive knowledge of mathematical puzzles or riddles and their solutions. These puzzles are called 'kuttaka', which is the name of a mathematical technique, whose oldest description is found in the Aryabhatiyam of the 5th Centruy CE. The 'kuttaka' is an important and powerful technique, which led to important developments in Indian Mathematics. Brahmagupta, in the 6th Century CE referred to algebraic techniques in general as '*kuttaka ganitha*'. The Mushari puzzles, which involve the solution of equations, may preserve a connection to this deeper tradition of Mathematics. It is intriguing that such knowledge exists among a community which is very low in the social hierarchy. We need a better understanding of the cultural transmission of mathematical knowledge between communities at different social strata. Culture can support the reproduction and circulation of mathematical knowledge not just through work, but also, as the puzzles indicate, through play. The revival of traditional art forms like music and their reshaping through digital technologies point to the possibilities of connecting art and Mathematics that are still to be explored.

Viewing the relation between 'everyday' and formal Mathematics through a different lens shows that political considerations are also relevant. As several writers have argued, with the growing dependence on mathematical science of modern technological societies, there is an increasing withdrawal of Mathematics to more hidden layers distant from everyday life. Not only is the complex Mathematics that underlies technological devices inaccessible to a lay person, but even everyday commerce may become emptied of mathematical thinking. With reagrd to everyday finance, which is relevant to nearly everbody, technology seeks to make Mathematics redundant. Calculators, EMI tables for loans, and other aids function as black-boxes that replace reasoning and calculation. This results in deskilling, and also takes attention and interest away from the underlying Mathematics. In a small study that we did, we found profound lack of awareness among educated users about how the credit card system operates and such critical issues as the effective rate of interest. Thus the increasing mathematization of society is accompanied by the growing de-mathematization of its citizens. Since Mathematics is entrenched as an essential part of the school curriculum, it begins to serve a different social function - that of weeding out large numbers from obtaining any access to the Mathematics and science that decisively shape modern society.

The emergence of small-scale producation activities as a part of the informal sector, offers to poorer households a means of subsistence and resistance against the harsh impact of changes in the organized economy. One cannot resist drawing a parallel in the light of the discussion on de-mathematization. Against the increasing trend of de-mathematization, the emergence of Mathematics on the street or in the workplace is a counter trend that resists the complete exclusion of the under-privileged from Mathematics. Of course such emergence by itself has no power to provide access to significant Mathematics. But the institution of education can amplify this possibility; bringing everyday Mathematics into the curriculum may prepare the way for bringing more Mathematics to wider sections of society.

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Ten Great Mathematicians

— H Subramanian

Introduction

It is not an understatement to say that there are essentially two subjects to study, viz. language and Mathematics. Everything else becomes dependent on them eventually. All of science, including management and social sciences, borrow on Mathematical formulations for concepts. The reason is simple. A precise and concise expression is made possible within the framework of Mathematics. Actually, the abstract layers of certain structural behaviours are increasingly becoming visible in the context of challenging problems from time to time. In this respect, Mathematics is also taken as another language - a symbolic one. Which Mathematicians can be singled out to have induced this transformation?

On a personal note, let me narrate an event. I was presenting a result as a Ph.D. student in Madurai to a visiting renowned Mathematician Nathan Jacobson from yale University, USA. He listened and finally asked a two-word question "Then what?" I was totally nonplussed. Later, I found an eye-opener in his question. If a result just closes as an issue, it is not big deal. If it leaves a door open for further thoughts, it is a great contribution. Which Mathematicians can be recognized using this screener?

There are several worthy names to consider. It is a hard choice to select just ten persons in the history of Mathematics who have influenced its development and changed the perception of the subject over time. My personal choices are Euclid, Fermat, Newton, Euler, Lagrange, Gauss, Cauchy, Riemann, Hilbert and the nom deplume Bourbaki. There are several geniuses who have missed the hit list. One may wonder why I have not included, say, the German physicist Albert Einstein (1879-1955), the Russian Mathematician Andrey Kolmogorov (1903-1987), the Indian Mathematician Srinivasa Ramanujan (1887-1919) or some ancient Indian Mathematicians. Einstein will find his place as one of the ten great physicists as he changed the perceptin of physics by his relativity theory. Kolmogorov is recognized as the founder of axiomatic probability theory influencing a lot of stochastic methods; but he does not spell a structural influence in Mathematics. Ramanujan is outright marvelous for his strong intuitive contributions; however this genius does not spur a change of perception of Mathematics. As there will be a separate article on ancient. Indian Mathematicans, I am not considering them.

Let me take up the ten cases now. I have chosen to list them in the order of the years theylived. In my opinion, the follower in the list was, at least indirectly, influenced by the work of the earlier ones.

> ***It is not the final result of Fermat's Last Theorem by itself that is great; but the techniques that have been developed to get a correct proof for it makes it stupendous.***

1. Greek Mathematician Euclid (around 300 BC)

The first axiomatic treatment in Mathematics was started by Euclid. His structure of proofs in geometry based on postulates about points and lines is a major conceptual contribution. His division algorithm for natural numbers is the most used one and the general Euclidean domains in future Mathematics emerged on that principle.

It is important to assess Euclid's achievements in the historic context of what was perceived as Mathematics before his time. It was a folklore understanding that it is all about practical computations in arithmetic and measurements of geometrical objects. Since around 1700 BC, numerical, algebraical and geometrical methods are attributed to Babylonian Mathematics. Later, a level of practical arithmetic and mensuration developed in Egypt and Italy. Much later approximately in the 7th to 6th centuries BC, we recognize the Greek Pythagorus and his contributions to geometry. Further on, Hippocrates is credited for circular arcs and areas; and, the logical thinking Zeno (remember his paradox!) introduced the concept of divisibility into infinite parts. During 430-349 BC, Plato's directions into philosophy dominated the conceptual frame of thought. Aristotle (384-322 BC) overlapped Plato's time with his say in formal logic and algebraic notation. By early 4th century BC, ideas of irrational numbers, geometric formulation of areas, algebraic procedures and even integration took roots.

It was Euclid's Elements (see Heath [Hea56]). Appollonius's conics and Archimedes's Analysis that has been recognized as serious Mathematics for centuries. The condification of geometry can be attributed to Euclid. We can say that Euclid, Archimedes and Appollonius made one great age of Mathematics.

It must be said that a relevant and practical Mathematics was persented in abstraction by Euclid. Such abstracting process has gained currency to transform Mathematics to unbelievable levels over time.

Descriptive geometry as well as algebra dominated the thinking during the first few centuries of AD. Menelaus, Ptolemy and Pappus developed synthetic geometry and Diophantus carried the banner of algebra, influenced by the Oriental thought.

From time immemorial to around 12th century AD, the growth in Mathematical thinking can be felt in terms of (i) computational arithmetic transforming to algebra and (ii) measuring geometrical entities (including studies in astronomy and spherical trigonometry) changing to the synthetic version of geometry like the ingredients of projective geometry. What happened during the next 400 years in mostly unknown. It mostly pertains to the contributions by Indians, Arabs and Greeks passing into the western world of Europe.

As it happened later in history, geometry went beyond Euclid's postulates. Geometry, as learnt today, is enmeshed in algebra. The key was the coordinatisation of geometry. The credit for this goes to Rene Descartes (1596-1650) (see Descartes [Des37]). For the next 200 years, significant achievements in Mathematics were focussed mainly on number theory and analysis. But Riemannian geometry (1894) brought another realm of geometric focus. Some other developments in geometry happened in the 19^{th} century. Along with some significant revelations in foundations through set theory, a thought provoking model of non-Euclidean geometry surfaced. The Russian Mathematicians Nikolay Ivanovich Lobachesky (1792-1856) and the Hungarian Mathematician Janos Bolyai (1802-1860), during the years 1826-1823, questioned Euclid's parallel postulate and obtained the geometric models without this postulate.

2. French Mathematician Pierre de Fermat (1601-1665)

Fermat was the great number theorist. Without going into all his work, especially on prime numbers, let us onlyh touch upon his remarkable scribbling in the margin space of his copy of the book by Diophantus. He mentioned that the space was insufficient to write his proof about the impossibility of a nonzero integer solution of the diophantine equation $x^n + y^n = z^n$ for $n \geq 3$ unleashed a significant trend thereafter in number theory and algebra to unravel his intended proof. This is familiarly known as Fermat's Last Theorem. The German Mathematician Emst Kummer (1810-1893) gave a false proof of this result based on a mistaken understanding of a certain factorization of $x^n + y^n$ as irreducible and unique. But he corrected his mistake soon and laid the seed for the modern ideal theory in rings.

Many other Mathematicians like Euler, the French Mathematician Adrien-Marie Legendre (1752-1833), the German Mathematician Johann Peter Gustav Lejeune Dirichlet (1805-1859) and Cauchy tried in vain to reconstruct the apparent proof that Fermat might have intended. They succeeded only on the cases n = 2; 4; 5 of Fermat's Last Theorem. With the advent of computers, most sophisticated computer methods were harnessed to verify this result; but they did not succed. Fermat's work was actually published posthumously in the year 1679. A settlement on this issue took about 400 years. We refer to Edwards [Edw77] and An drew Wiles [Wil95] to appreciate the vast realm and technicality of Mathematics that it createed on this matter.

It is not the final result of Fermat's Last Theorem by itself that is great; but the techniques that have been developed to get a correct proof for it makes it stupendous.

3. British Mathematician Sir Isaac Newton (1643-1727)

Newton was the most original contributor among the 17th century Mathematicians. The binomial theorem for rational exponents led to the ideas of infinite series. Obtaining areas by the method of summation of infinite subtparts can be attributed to many people, Archimedes onwards to the British Mathematician John Walls (1616-1703). The idea of differentiation goes back to the French Mathematician Blaise Pascal (1623-1662), who is credited also for the invention of digital calculators. In this background, both Newton and the German Mathematician and philosopher Gottfried Wilhelm Leibniz (1646-1716) attempted the fundamental principle of calculus, viz. the inverse process of differentiation is integration. Leibniz got to calculus independently during 1673-1676 from the works of the British Mathematician Isaac Barrow (1630-1677) and Pascal. Newton got to calculus from the ideas of his teacher Barrow as well as Fermat and Walls. Both Newton and Leibniz were unable to establish calculus on sound logical basis. Newton included in his work (see Newton [New87]) a short offhand explanation of calculus. This was the greatest of all scientifc treatises; in it can also be found the greatest of scientific treatises; in it can also be found the greatest of scientific generalizations, viz. the law of gravitation. Newton's aim was to understand nature.

Newton later on explained his calculus on rates of change. In this process, he used the binomial theorem for integral exponents and generalized the same for rational exponents. Leibniz used the idea of infinitesimally small differentials, denoted by dx and dy and tried (unsuccessfully) to explain his method in terms of sums and quotients of these. We are using this notation even today. The method of differentials became the mainspring of Mathematical developement duringthe course of 18th and 19th centureis AD. Leibniz's influence on the European continent was far greater than Newton's. Continuity of functions was an intuitively accepted phenomenon in the earlier rounds of thinking. Fortunately for the progress of Mathematics, Newton and Leibniz took for granted that all functions have derivatives. It was difficult enough to obtain an acceptable notion about it. To the differential calculus thus discovered was added the opposite contruction of the integral calculus; and the work of Archimedes was belatedly rediscovered. After sometime, it was realized that certain functions might be represented by power series also. Mechanics, even for Newton, led to considering functions as integrals of differential equations. The infinitesimal geometry of curves, extended to surfaces, demanded the introduction of equations with partial derivatives, necessary also for mechanics of vibrating cords. A study of periodic phenomena led to the consideration of trigonometric series. The original classic by the French Mathematician Jean-Baptiste-Joseph Fourier (1768-1830) (see Fourier [Fou22]) contains the basic ideas in this regard, although with no rigour. Thus each forward step in Mathematics engendered, by a chain of creations, the introducation of new entities which were used as tools for other studies and other creations.

Newton's contribution changed the face of both Mathematics and physics.

4. Swiss athematician Leonhard Euler (1707-1783)

The Bernoulli family from Switzerland, running through mid 17^{th} century to late 18^{th} century, were fired with enthusiasm for dirrerential calculus. Euler was Johann Bernoulli's student. He, in his treatise (see Euler [Eul48]), brought the concept of function and infinite processes into analysis. Euler was one of the founders of calculus of variations and differential geometry. Both are applications of differential calculus, to cases in which a function depends on another function or curve in calculus of variations and to general properties of curves and surfaces in differential geometry. After Fermat, Euler was the greatest number theorist. The Euler totient function has its claim of importance in number theory. It extended a Fermat result on prime numbers to nonprime numbers also. Euler's formula in radicals for quartic equations is the lst such case because Abel proved later that it is impossible to obtain such formulas for fifth and higher degrees. His method of exeraction of square roots modulo prime numbers and continuation of certain work by Fermat is another contribution that led Gauss to explore sitll further. He made strides into continued fractions. In 1779, he also posed a certain conjecture on orthogonal Latin squares; but it took nearly 200 years to prove Euler wrong (see Parker [Par59} and Bose, Shrikhande and Parker [BS59] & [BSP60]). His calculus of quotients of qualitative zeroes and operating with sums of divergent series created furore. Euler also contributed to Mathematical physics.

While Greeks put everything side by side, all aspects of real numbers did not merge with the Greek heritage. While this is so, it seems that the later developments from the beginning of 17^{th} century happended through a perception of compelling need to expand, analyze and rationalize various aspects together. Euler is to be credited for his wide-spread contributions in Mathematics, though he was seen to lack soundness in logical foundations. After Newton and Leibniz, the pioneering 18th centruy Mathematician was Euler.

5. Italian-French Mathematician Joseph-Louis Lagrange (1736-1813)

Born in Italy, Lagrange had a carrer in Prussia and later moved to France. In the context of the French Revolutionary period, we note the establishement of Ecole Polytechnique in the year 1794. Lagrange and Pierre-Simon Laplace (1749-1827) were its first teachers. Lagrange's contribution is in a variety of subjects - algebra, number theory, analysis and mechanics. His interpolation formula is one of the most applicatble results in numerical analysis. The result in group theory known today as "the order of an element in a finite group divides the number of elements in the group" is attributable to his work extending Euler's congruence (see on the preceding page) that absorbed Fermat's congruence. He continued Euler's work on continued fractions. He discovered a rule for existence of multiple roots of a polynomial based on the greatest common denominator of the polynomial with its derivative. He is the cofounder, with Euler, of calculus of variations. In his works (see Lagrange [Lag88] & [Lag97]) on mechanics and analysis, he uses deductive logic to bring mechanics in the framework of rigorous Mathematical analysis. Lagrange's theory of functions, the graphical representation of complex numbers obtained in 1813-1814 by the Swiss Mathematician Jean-Robert-Argand (1768-1822) and the imaginary period of elliptic functions explained by Abel in the year 1824 led Cauchy to the so-called integral theorem continuation of Euler with a difference. He brought a level of abstraction that paved a smoother path for future Mathematicians.

6. German Mathematician Carl Friedrich Gauss (1777-1855)

Gauss was one of the first to feel the need for rigour in Mathematics. In the year 1796, at the age of 19, he studied the Euclidean constructions in geometry; arrived at the notions of contructible numbers; and, created a setting for all algebraic numbers of certain types to be considered. The phrases used today like Gaussian integers, Gaussian channel acknowledge his remarkable contributions in related areas. In 1799, he proved that every polynomial equation with complex coefficients has a root. In the year 1801, he contributed (see Gauss [Gau01]) to number theory in terms of theory of congruences and quadratic reciprocity; these are enormously significant results. His quadratic reciprocity law enabled many numerical calculations without effort. He conjectured that for infinitely many prime numbers p, p-1 is the least number such that p divides 10^{p-1} -1. This conjecture is still unresolved. He provided the basic result, known today as Gauss Lemma, that enables construction of unique factorization domains. He contributed (see Gauss [Gau28]) to differential geometry exploiting the parametric representations.

It is often the case that serious researchers today look backwards to Gauss for inspiration. Another reason for them to do this is to make sure that this giant has not already introuduced their ideas.

7. French Mathematician Augustin Louis Cauchy (1789-1857)

It was Cauchy who succeeded in introducing clarity and rigour. He was "forced to accept propositions which may seem a little hard to accept; for example, that a divergent series does not have a sum". He introduced, with precision, the necessary definitions of limit, of convergence, and thus made possible, in a short time, great advances in areas which had been finally clarified.

Actually at the beginning of 19^{th} century, Cauchy closed one period in the history of Mathematics and inaugurated a new one which would appear to be less hazardous. He ruthlessly tested the product of three centuries, establishing an order and a rigour unknown before. He rejected as too vague the habitual appeals to "generality of the analysis" and determined the conditions of validity of statements in analysis with rigorous definitions of continuity, of limits, of different sorts of convergences of sequences or series, which he provided. In the early 19^{th} century, Abel described the state that Mathematics was in when he entered it thus : "Divergent series are completely an invention of the devil, and it is a disgrace that any demonstration should be based on them. One can draw from them whatever one wishes when they are used they are the ones that have produced so many failures and paradoxes. Even the binomial theorem has not yet been rigoursly demonstrated. Taylor's theorem, the base of all higher Mathematics, is just as poorly established. I have found only one vigorous demonstration of it that of Cauchy."

In the year 1825, Cauchy proved the well-known integral theorem of complex functions. At the core foundation level, he offered a construction of the real number system. Also constructed the real number system. Much later in the year 1895, the German Mathematician George Cantor (1845-1918) formulated (see Cantor [Can95]) the theory of sets and introduced how to reason in a framework. In doing so, he justified Cauchy's construction of real numbers to be the same as two other versions created by the German Mathematicians Richard Dedeking (1831-1916) and Kari Weirstrass (1815-1897). We refer to the complete works of Cauchy in [Cau74] for his detailed contributions.

We can say that Cauchy and Cantor started the age of reason. It is Cauchy who laid the strong foundation of Mathematics by insisting on logical reasons to prevail at every level.

8. German Mathematician Georg Bernhard Riemann (1826-1866)

During the mid-19^{th} century, Riemann ruled the development of future Mathematics beyound the imagination of othose times. One of his masterpieves of work is complex function theory. His geometric intuition in complex analysis in terms of conformal mappings and the so-called Riemann surfaces, his method of handling differential form for arc length, curvature tensor etc. (in the year 1854) involving ideas found useful and essential later in general relativing ideas found useful and essential later in general relativity and his conjecture (in the year 1859) that is still unresolved going by the name Riemann Hypothesis and which has unleashed an unsurpassed level of Mathematical developements - all these qualify him as the undisputed master and genius.

Riemann's Hypothesis remains one of the most intriguing conjectures in all of Mathematics. It is difficult to describe it without going into some Mathematical vocabulary. It states that all the nontrivial zeroes of a certain complex - valued function of a complex variable, described in terms of an infinite series, must have real part euqal to 1/2, The subject matter of this unresolved conjecture has triggered valuable research in analytical numbers theory and complex function theory. Riemann Hypothesis is still unproved. But serveral thousands of zeroes have been verified to fall in place by use of computers.

The Riemann Hypothesis, if found true, would have enormous consequences in number theory. For instance, it would establish a better handle on the nature of distribution of prime numbers. No one would have thought about a connection between prime numbers and analytic functions.

We refer to Riemann's works (see Riemann (Rid90]). We also refer to Laugwitz [Lau99] to find how Mathematics changed since Riemann.

9. German Mathematician David Hilbert (1862-1943)

The German Mathematician David Hibert (1862-1943) provided the steering wheel for most of the Mathematical developments during the 20^{th} century by his 23 famous problems presented in his address to Second International Congress of Mathematicians at Paris (see Brouder [Bro76]). The surge in ideas contributed by these problems was huge. Though these challenging problems developed furture Mathematics to a great extent, some of his expectations were belied later. A notable one was his own student Godel's result (see Godel [Go40]) in logic proving that arithmetic cannot be simultaneously consistent (meaning both a statement and its oppisite cannot be true) and complete (meaning either a statement or its opposite is true); that was contrary to Hibert's intuition. Hilbert's contribution, in the year 1906, to the theory of infinite dimensional spaces is immense. Hilbert is remarkable for his conceptualizatin of certain earlier trend - setting problems. In the second half of 19^{th} centrury, problems in electrostatics and potential theory led to a study of integral equations. In the year 1877, the United States Mathematician George William Hill studied matrices of infinite size relating them to perturbation theory of lunar motion. And, in the year 1900, Ivar Fredholm discovered the algebraic analogue of the theory of integral equations. Henri-Leon Lebesgue laid the ground work with measure theory for the later contributions by Banach and Frechet to generalize Hilbert's work. In the year 1922, the Austraian - Hungarian (later territorially Polish) Mathematician Stefan Banach studeied the aspects of "geometrizing" the spaces. And the French Mathematician Maurice-Rene Frechet, in the year 1928, generalized these further. Hilbert's contributions in commutative algebra, diophantine euqations, number theory etc. are equally noteworthy.

Hibert's list of problems kindled so much of research activity during the first five decades of 20th century that this peirod is known as the golden age of Mathematcis.

10. Nicolas Bourbaki (1935-continuing)

A group of Mathematicians (mainly French) go together under this pseudonym. Nicolas refers to an ancient Greek hero from whom a French General Charles Soter Bourbaki apparently descended. When Andre Well was a first year student in Ecole Normale Superieur. He attended a lecture by a senior student, who mockingly presented false theorems and attributed them to various French generals. The last and most ridiculous theorem was named after this Bourbaki. Andre Weil, Henri Cartan, Claude Chevalley, Jean Dieudonne and a few others, all young under 30 years of age, were passionate to bring about changes in future currinulum of Mathematics. Their thought was to publish books from a conceptual and fundamental standpoint. The idea of a forum was born and it was named after Bourbaki, with humour intended.

The first Bourbaki Congress was in the year 1935. The group decided on certian rules for themselves. Membership is limited to 9. Each member would retire at the age of 50. They would meet three or four times a year, each time for a week or two, for a total discussion on the various projects. All members should participate to the same extent that a member is expected to. They should use only axiomatic framework and structural classification of Mathematics to write the books that are agreed to be written. No references could be cited other than a Bourbaki book in line. Everyone would prepare a presentation of a topic or a chapter for the book and all others would discuss the material in entirety. Together they would arrive at the final version. There is no authorship and the publishing will go under the name Bourbaki. Bourbaki's aim was to publish high-class text-books quickly. The target size of a book was about 1000 pages running into approximately 10 chapters with the target time set for about 6 months. Their initial list of books is :

a) Book I. Set Theory

b) Book II.Algebra

c) Book III. Topology

d) Book IV. Functions of One Real Variable Variable

e) Book V. Topological Vector Spaces

f) Book VI. Integration

The project moved slowly; only certain chapters were completed by 1942 because of World War II and members going abroad. Roger Godemont, Jean-Pierre Serre, Pierre Samuel, Laurent Schwartz, Samuel Eilenberg were recruited as new members. By 1958, the books were completed. By then, some founding members ste4pped down and Alexander Grothendieck, Serge Lang, John Tate joined.

Meanwhile Mathematics had grown to a considerable extent, also owing to the inuence of Bourabaki. Members felt that they were not universal Mathematicians to join in all the book projects. However, the decision of Bourbaki was even if not universal, their interest to participate in that, everything was mandatory. No membre could stay on the principle of their speciality contribution only. About the rigidity of linear order of arrangement of the topics in books, compromise was made that an organic development would be acceptable without disrupting the unity of Mathematics and structural aspects. The earlier six books needed revisions so new projects could build on them. Bourbaki carried out these and also completed by the year 1980 quite a few chapters in three more books, viz.

a) Book VII. Commutative Algebra

b) Book VIII. Lie Groups and Algebras

c) Book IX. Spectral Theory

Yer another one (Differential and analytic varieties) completed by Bourbaki was just a summary of results and nto a full exposition of thougths. It was just to help the organization of other books.

The various titles of Bourbaki books given here are written in English. Actually, the titles and the work were in French. The first 6 books were later translated into English by Bourbaki.

Bourbaki also publishes several survey articles on advancede topics with their interntion to reach out these topics to nonspecialists. These titles emerge after intensive seminars held frequently.

Bourbaki certainly set standards for what a professional Mathematician should know. They brought out books aimed as text-books but they are more like reference books or encyclopedia-cum-treatise-cum-monograph or whatever. What will be the future of Bourbaki? Will their book porjects die? Will their seminars and publications take over the main Bourbaki engagement? Will specialists only prevail under Bourbaki banner? The basic starting reason for Bourbaki is the unity of Mathematics. History shows that survival.

Conclusion

Like I said in the beginning, language and Mathematics are the only two sujbects that are characteristically fundamental. Through them, we can handle whatever appropriate expressions are needed to convey ideas in essence (abstraction) and lead these ideas to explore further thoughts (concretisation). They have become essential and irreplaceable in our daily life. The selected Mathematicians in this article have amply fortified this and made future embellishments a distinct possibility.

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